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Belgian Nuclear Research Centre

Jens Peter Frankemölle, Johan Camps, Pieter De Meutter & Johan Meyers

HARMO 22 | 13 June 2024 | Pärnu

Near-range source term estimation and uncertainty quantification informed by an early warning network around a nuclear facility

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Towards a comprehensive near-field inverse modelling framework: gamma dosimetry, gaussian plumes and Bayes's theorem

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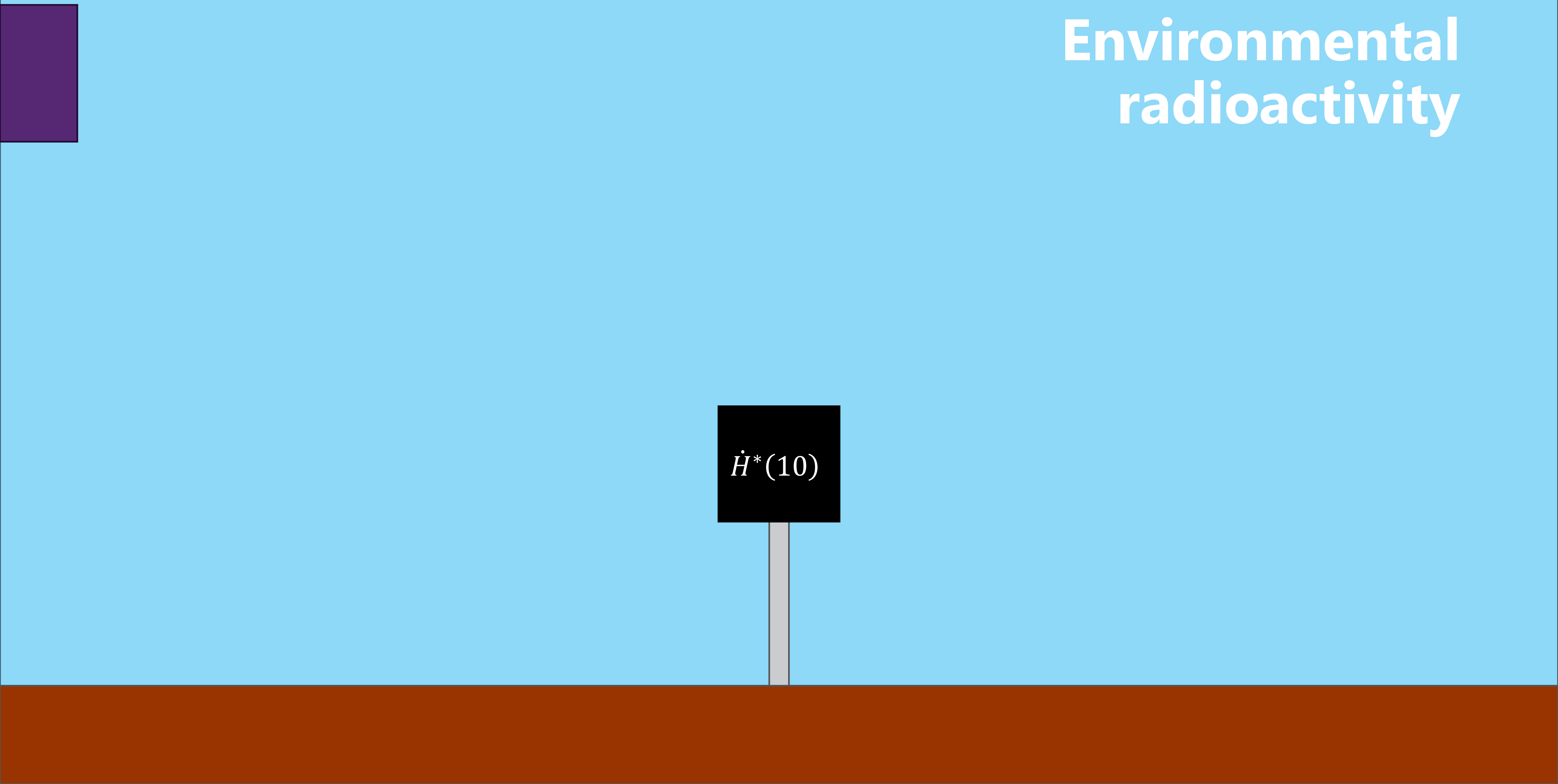
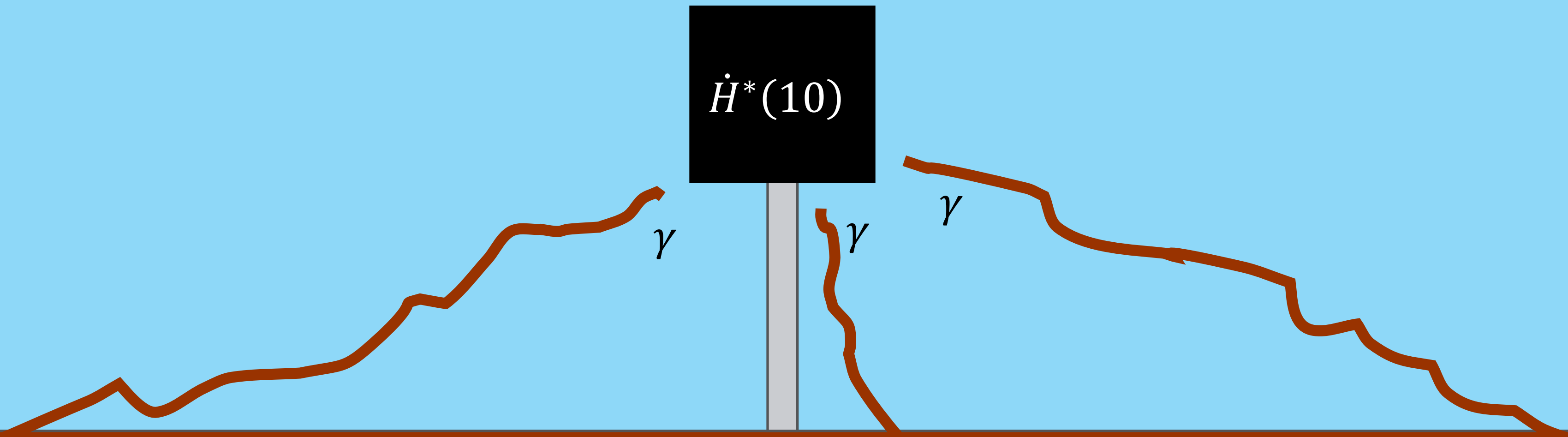
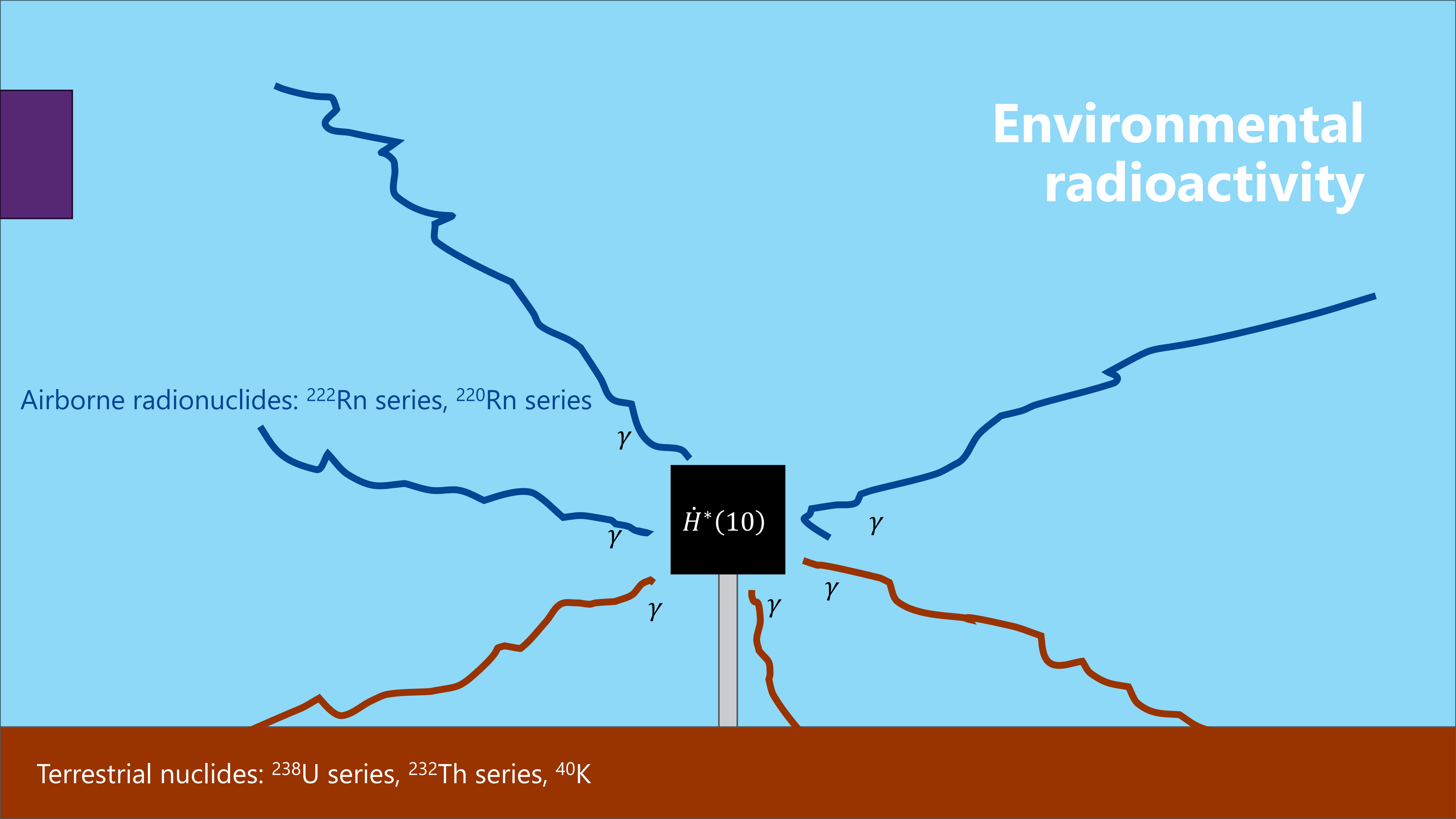
Environmental radioactivity



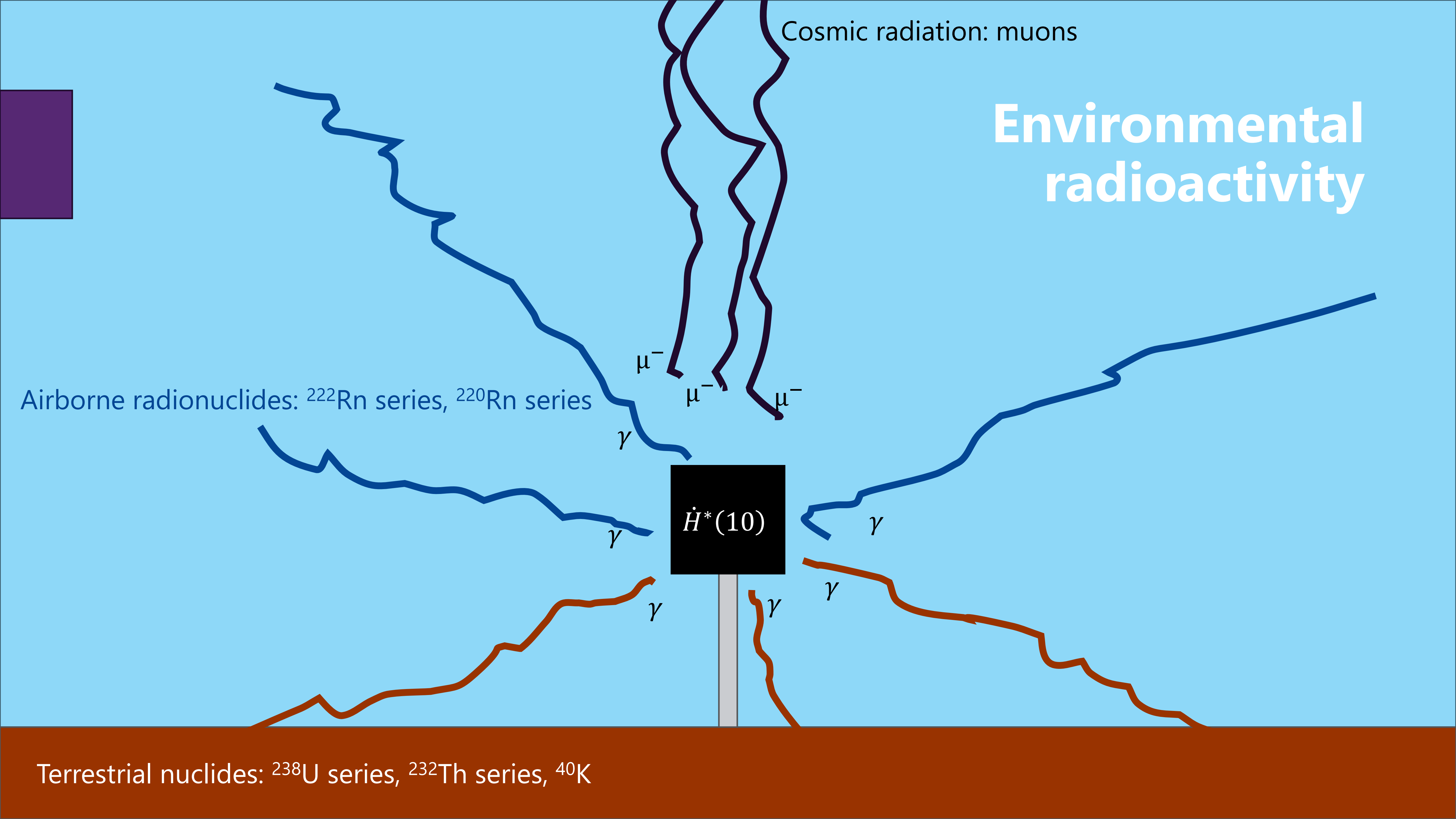
Environmental radioactivity

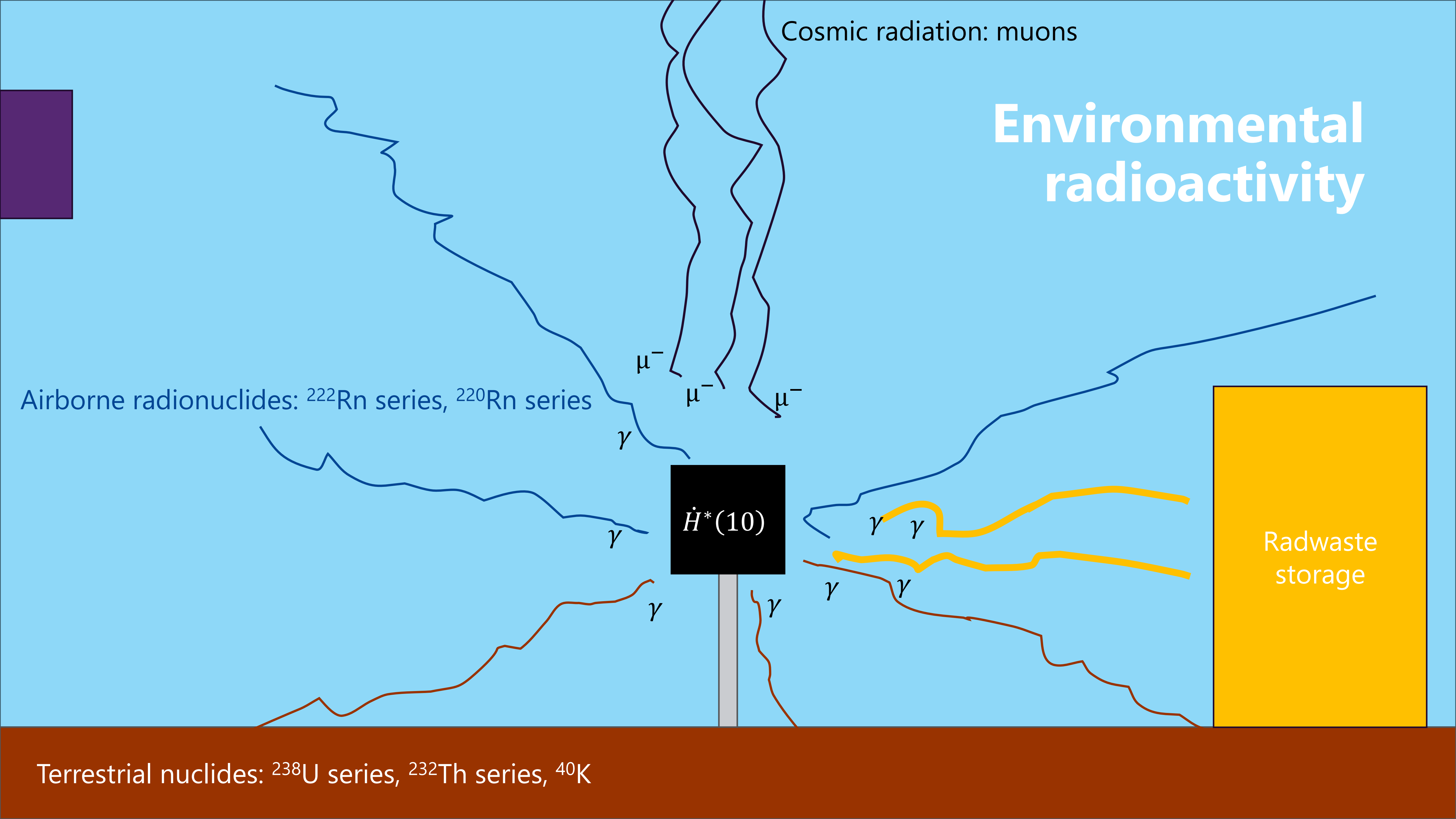


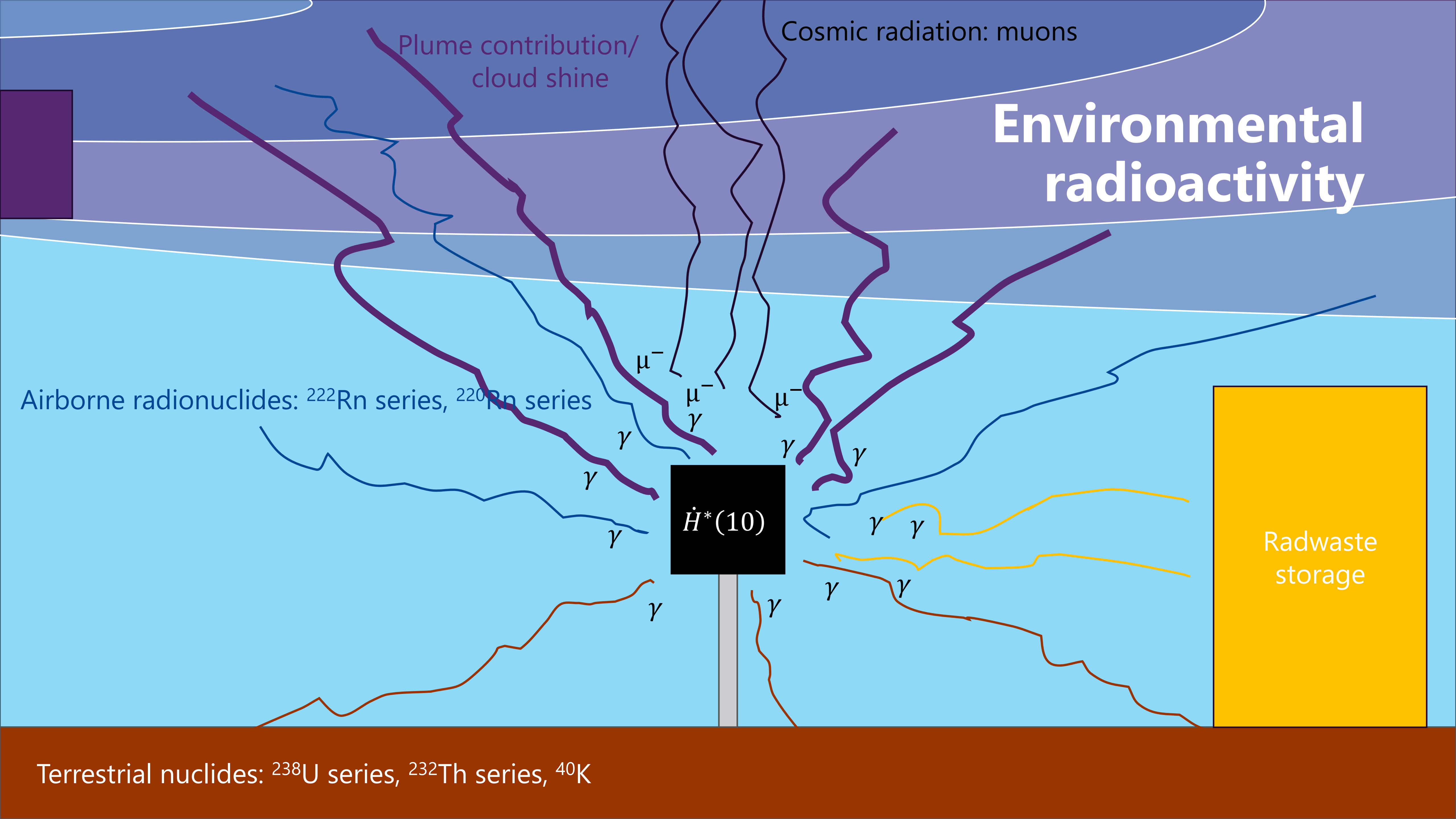
Terrestrial nuclides: ^{238}U series, ^{232}Th series, ^{40}K


Environmental radioactivity
Airborne radionuclides: 222Rn series, 220Rn series
γ
γ
γ
Ḣ*(10)
γ
γ
γ
Terrestrial nuclides: 238U series, 232Th series, 40K

Environmental radioactivity

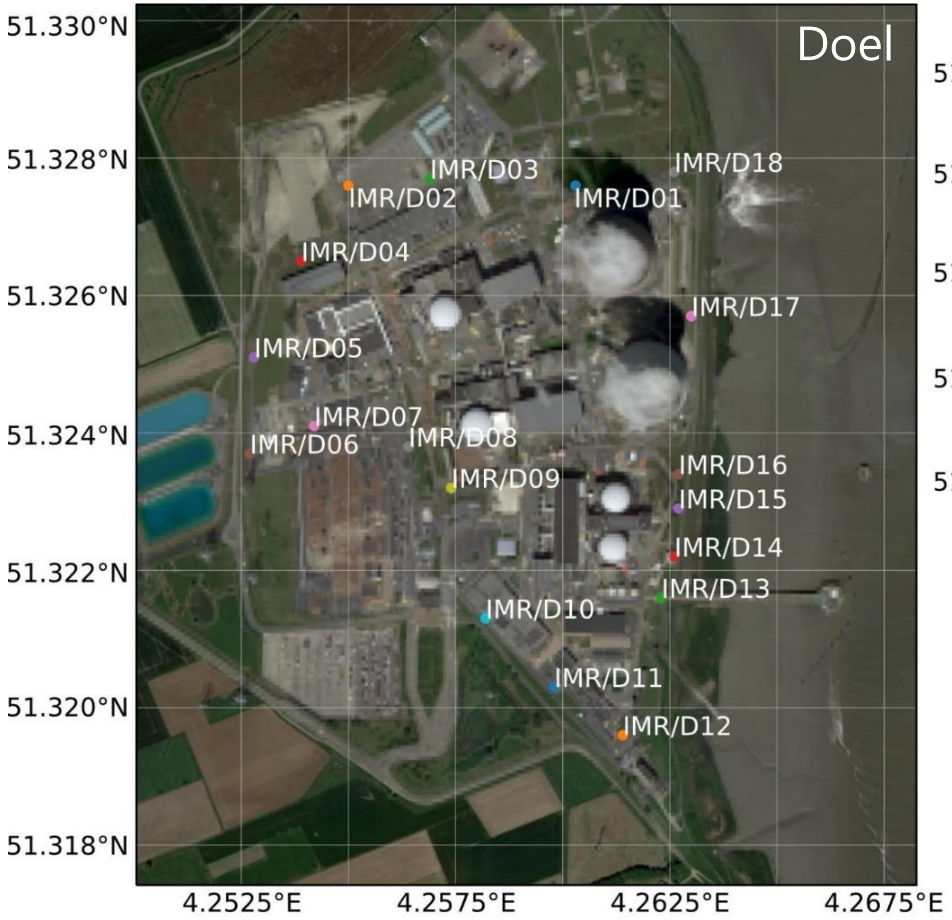


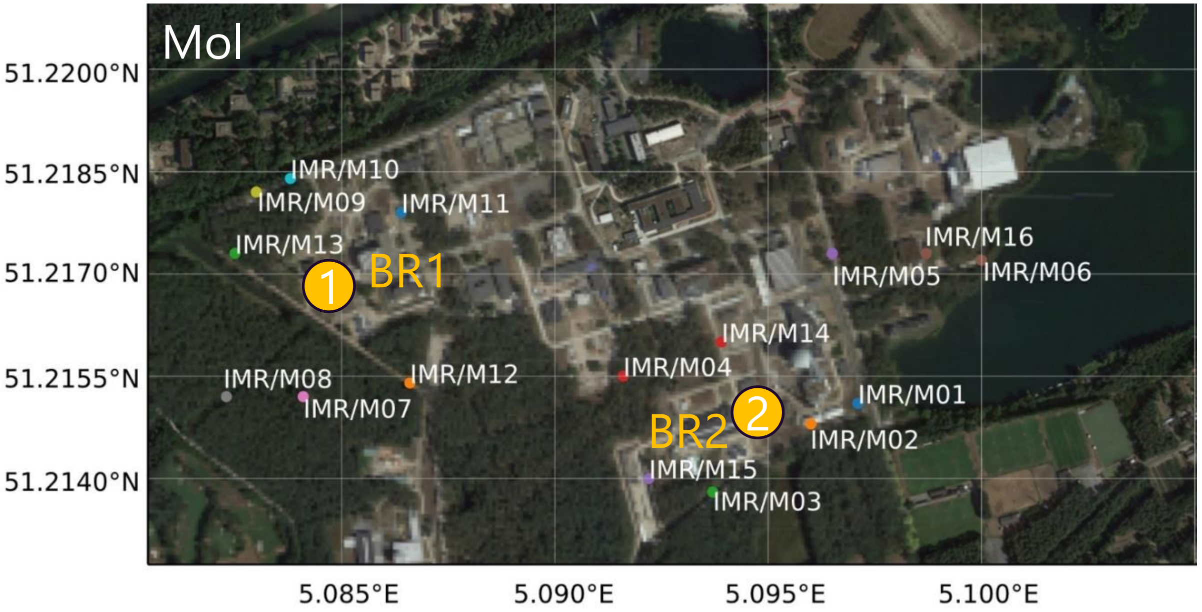

Environmental radioactivity
Cosmic radiation: muons
μ⁻
μ⁻
μ⁻
Airborne radionuclides: ²²²Rn series, ²²⁰Rn series
γ
γ
$\dot{H}^*(10)$
γ
γ
γ
γ
γ
γ
γ
Radwaste storage
Terrestrial nuclides: ²³⁸U series, ²³²Th series, ⁴⁰K

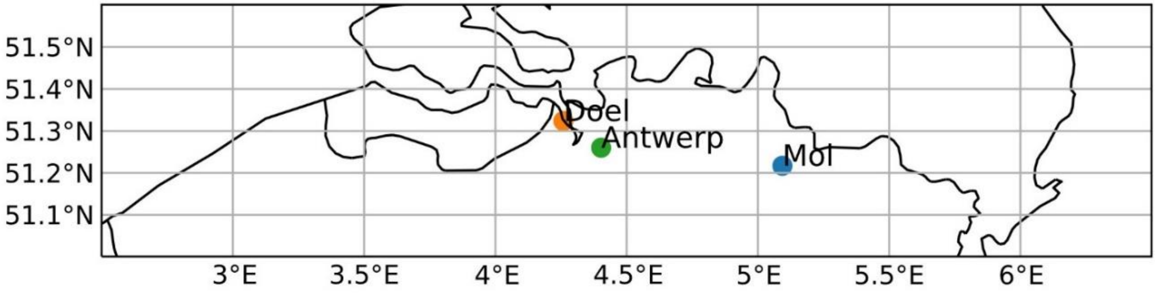


Cosmic radiation: muons
Plume contribution/
cloud shine
Environmental
radioactivity
Airborne radionuclides: ^{222}Rn series, ^{220}Rn series
μ^-
γ
$\dot{H}^*(10)$
Radwaste
storage
Terrestrial nuclides: ^{238}U series, ^{232}Th series, ^{40}K

Early-warning networks: ring stations







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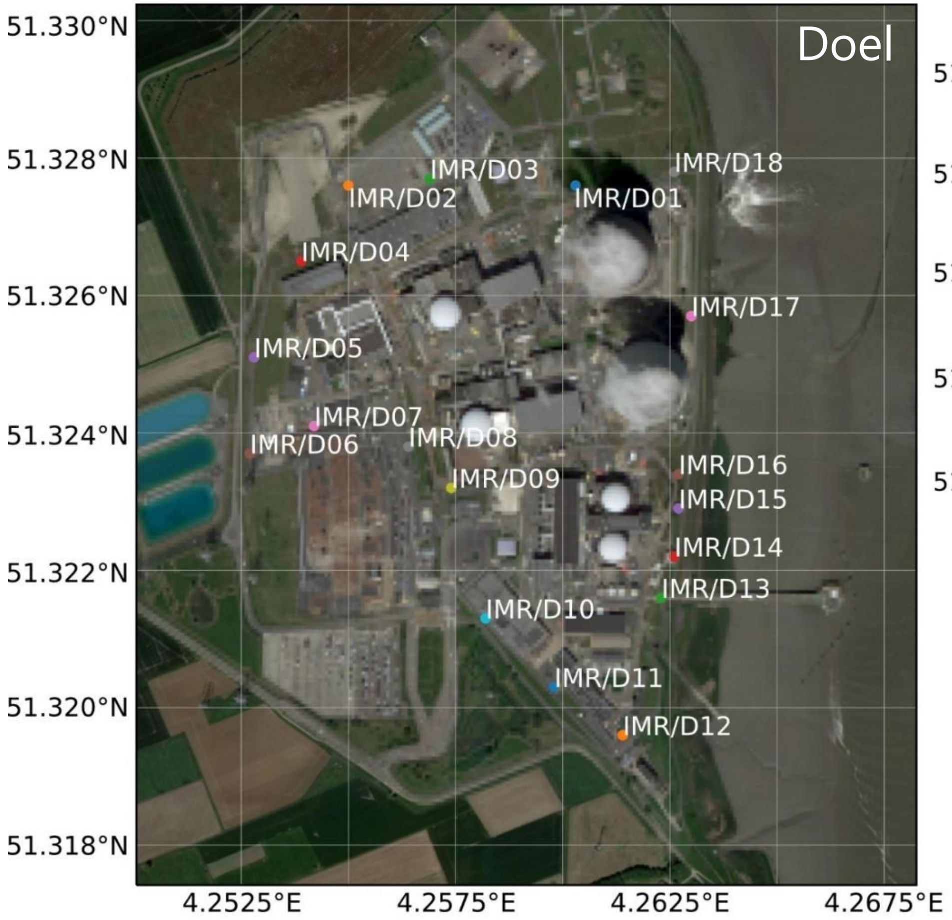

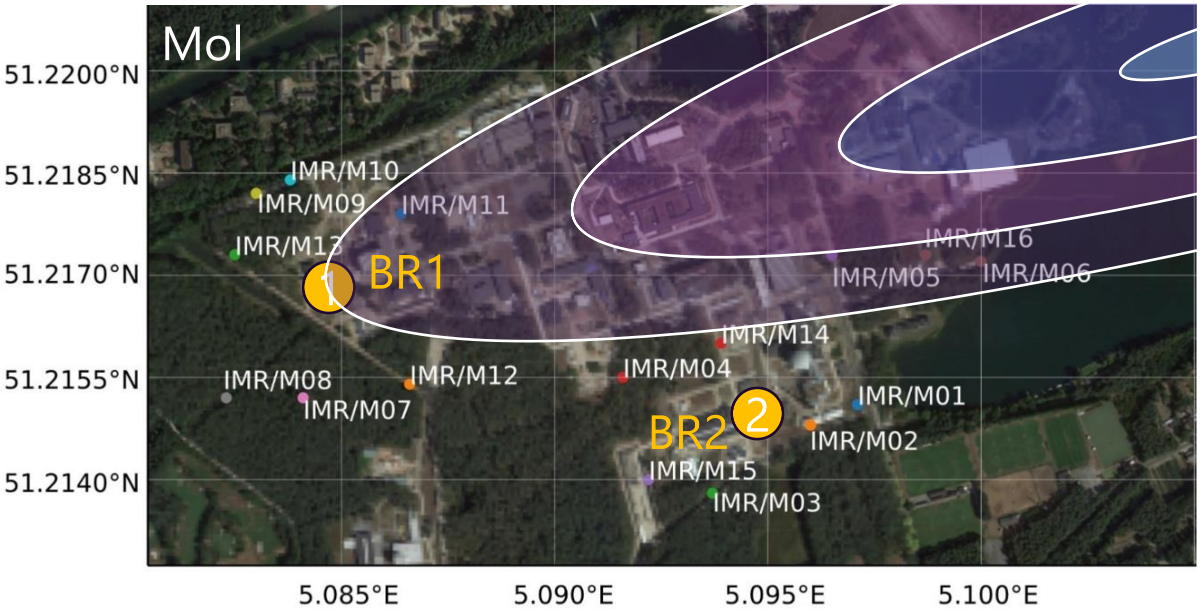

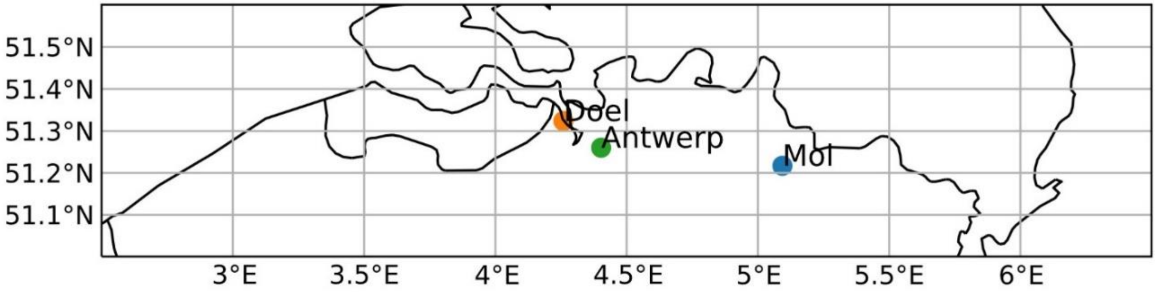

Comprehensive inverse modelling framework using Bayesian inference

$$f_{\vec{X}|\vec{Y}}(\vec{x}|\vec{y}) = \frac{f_{\vec{Y}|\vec{X}}(\vec{y}|\vec{x}) f_{\vec{X}}(\vec{x})}{f_{\vec{Y}}(\vec{y})}$$

Comprehensive inverse modelling framework using Bayesian inference

- Observations ($\vec{y}$)
- Formulation of the priors for $\vec{x}$ and likelihood ($\vec{x} \rightarrow \vec{y}$)

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  - Source term
  - Dispersion coefficients
- Uncertainty quantification $\mathrm{Var}[\vec{x}]$

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Comprehensive inverse modelling framework using Bayesian inference

A B C

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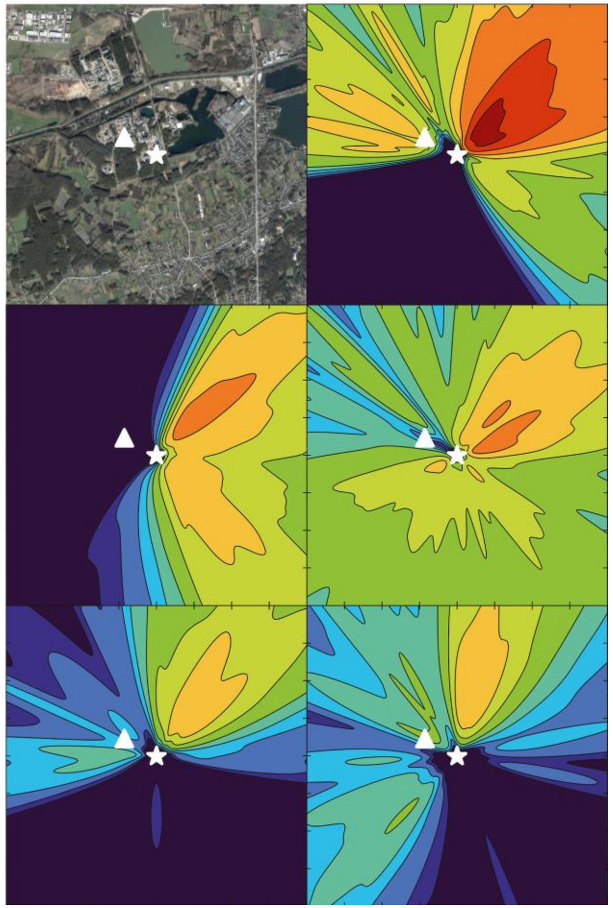
Atmospheric Dispersion & Dose Equivalent Rates
ADDER

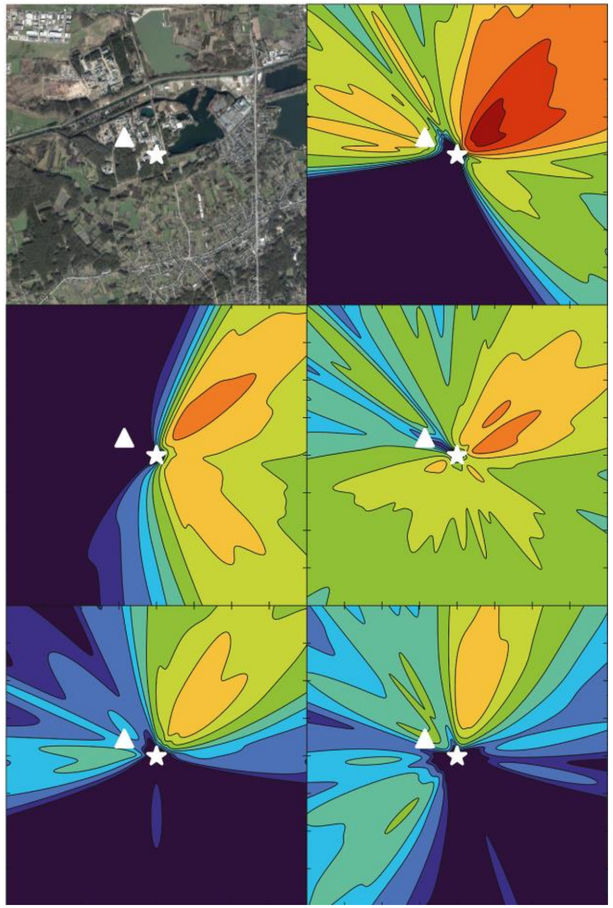


Frankemölle, J.P.K.W. et al. *J. Environ. Radioact.* **255**, 107012 (2022)

A

Atmospheric Dispersion & Dose Equivalent Rates ADDER

- Extended Gaussian plume model
  - Site-specific parameterisation
  - Ground and capping inversion reflection
  - Buoyant plume rise



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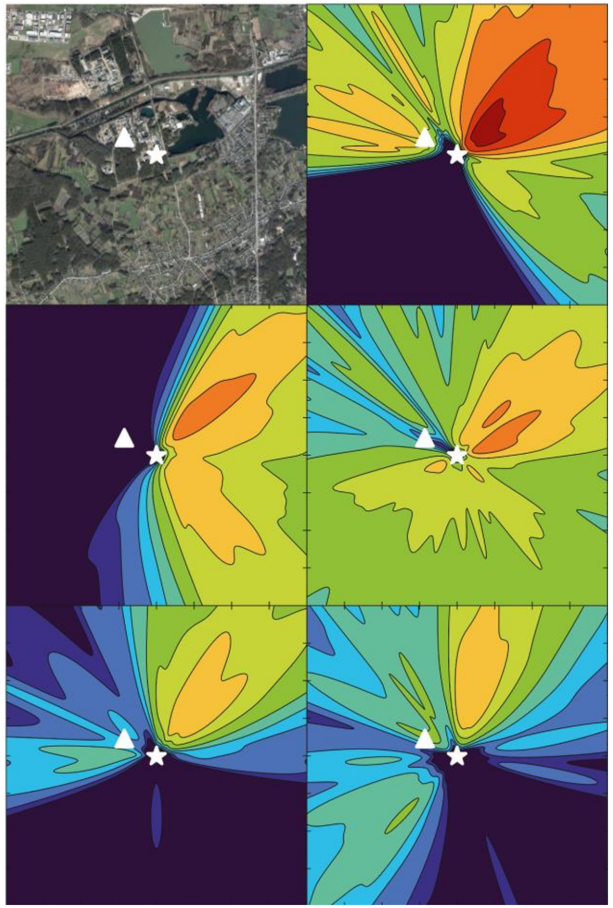
  - Air kerma and ambient dose equivalent rate
  - 3D-integrated (no dose coefficients)



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A Atmospheric Dispersion & Dose Equivalent Rates ADDER

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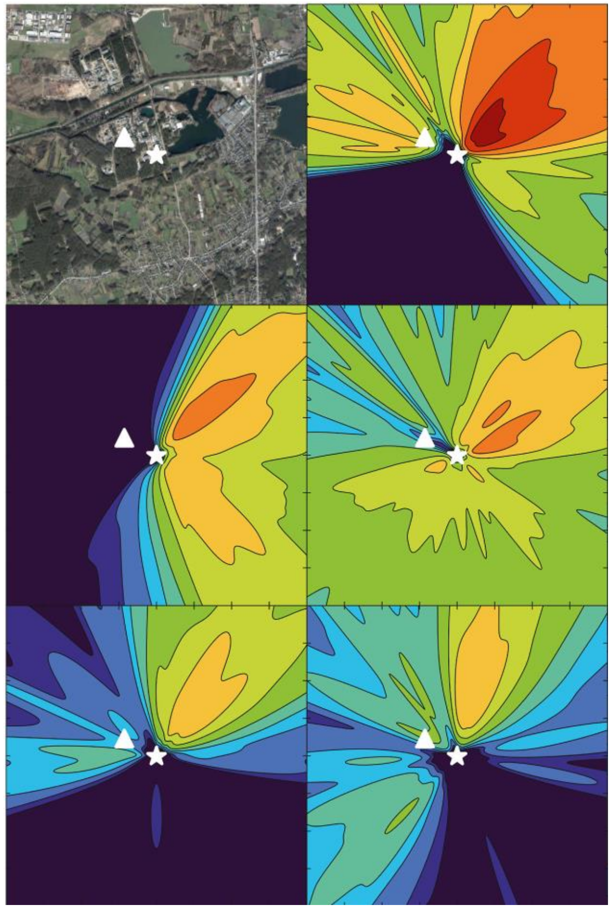
  - Air kerma and ambient dose equivalent rate
  - 3D-integrated (no dose coefficients)
- Currently extending it for out-of-the-box inverse modelling using PyMC framework



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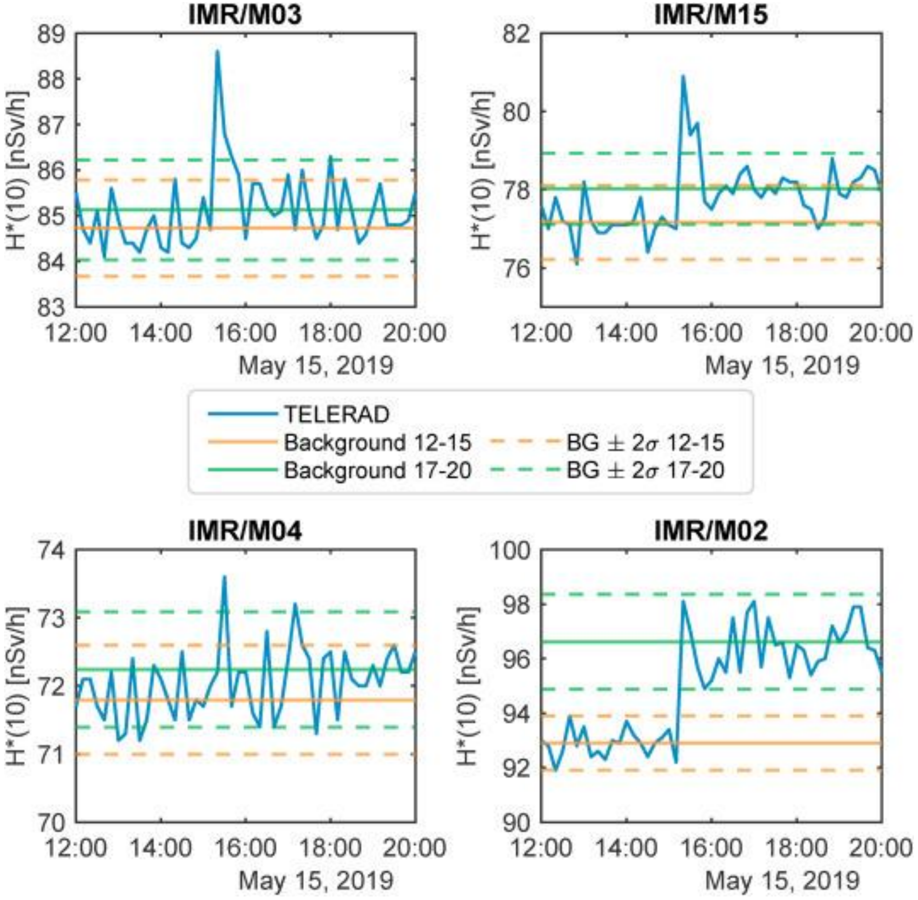
A

Simulating the 2019 ^{75}Se incident at BR2, Mol

ADDER



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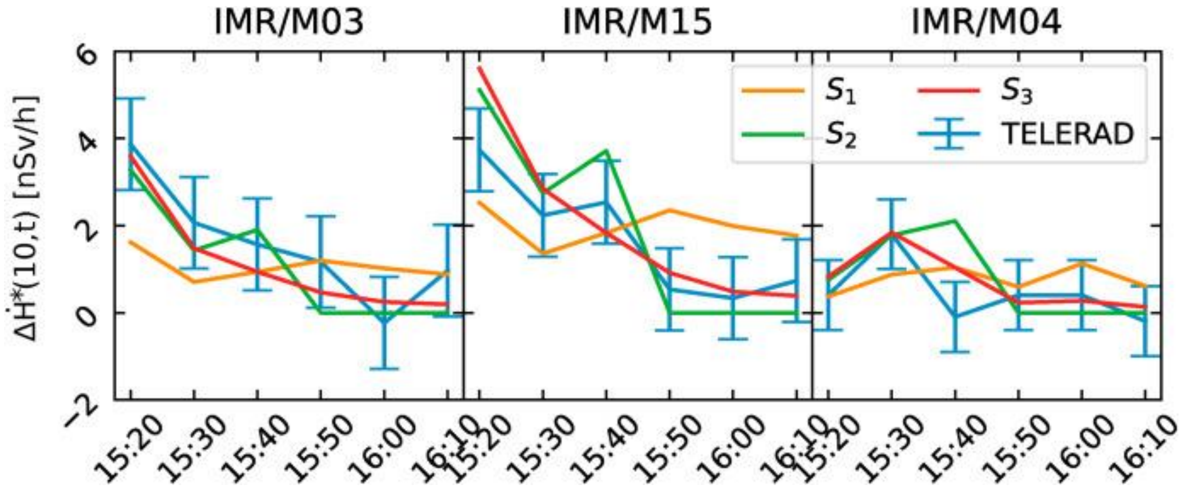

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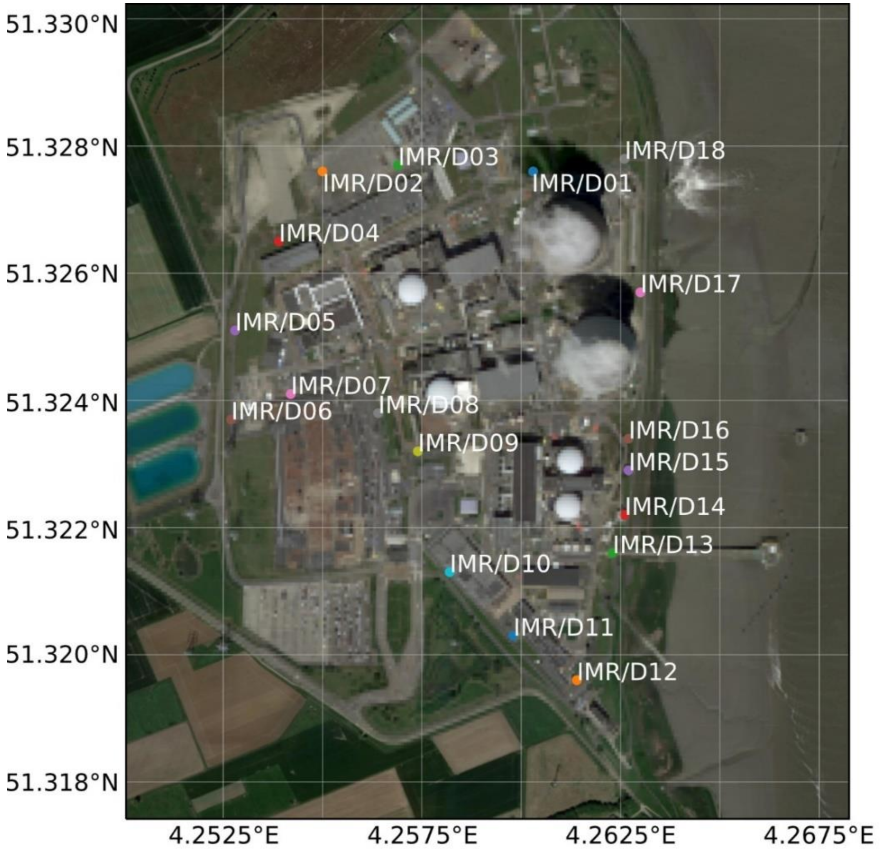
B Model for background radioactivity

- Stochastic representation of the background in a detector network with k detectors

$$\vec{\dot{H}} = \begin{bmatrix} \dot{H}_1 \\ \vdots \\ \dot{H}_k \end{bmatrix} \sim \mathcal{N}(\vec{\mu}, \vec{\Sigma})$$



Frankemölle, J.P.K.W. et al. *Submitted for publication* (2024)

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:hastic representation of the backgrounc
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$\vec{H} = \begin{bmatrix} \dot{H}_1 \\ \vdots \\ \dot{H}_k \end{bmatrix} \sim \mathcal{N}(\vec{\mu}, \vec{\Sigma})$
1g No U-Turn Sampling (PyMC)
alibration: estimate $\vec{\mu}$ and $\vec{\Sigma}$
ediction: estimate $\vec{\mu}_{\mathrm{u|o}}$ and $\vec{\Sigma}_{\mathrm{u|o}}$

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  - Calibration: estimate $\vec{\mu}$ and $\vec{\Sigma}$
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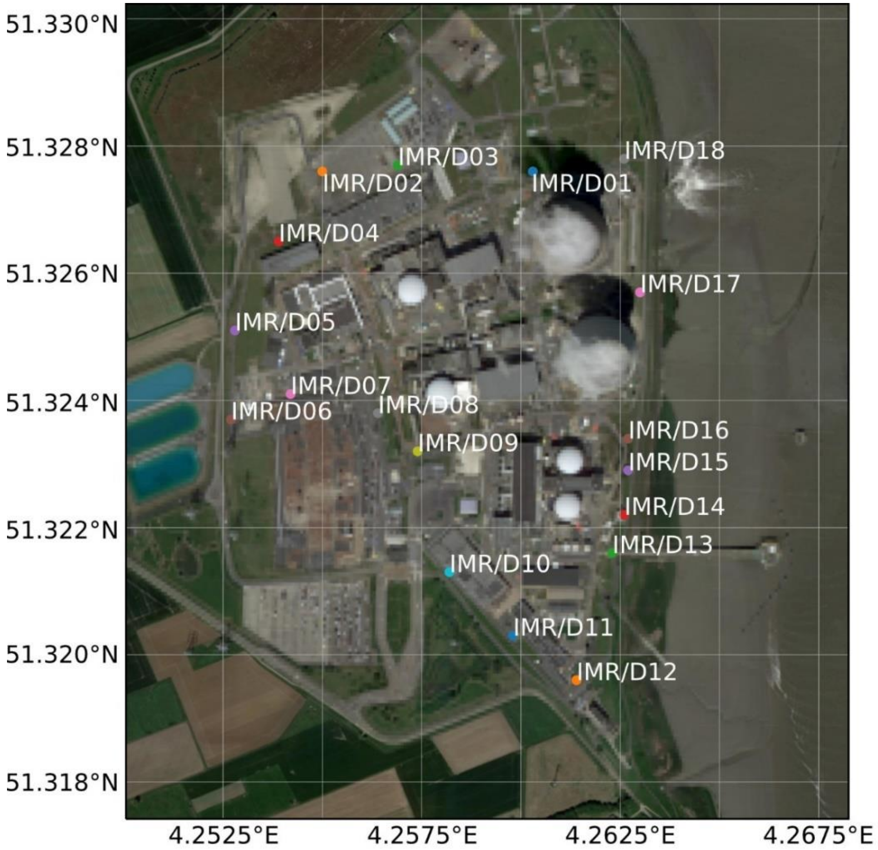

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B

Observations, likelihood and priors required to obtain the posterior $f\left(\vec{\mu}, \vec{S}, \mathbf{R} \middle| \vec{H}\right)$

- Observations
  - $\dot{\boldsymbol{\mathcal{H}}} = \left[\vec{H}_{t=t_1}, \dots, \vec{H}_{t=t_N}\right]$

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  - $\dot{\mathcal{H}} = [\vec{H}_{t=t_1}, \dots, \vec{H}_{t=t_N}]$
- Likelihood

$$f(\vec{H} | \vec{\mu}, \vec{S}, \mathbf{R}) = (2\pi)^{-\frac{Nk}{2}} \prod_{i=1}^{N} \left| \vec{S}\mathbf{R}\vec{S} \right|^{-1/2} \exp\left[-\frac{1}{2} \left(\dot{\vec{H}}_j - \vec{\mu} \right)^{\top} \left(\vec{S}\mathbf{R}\vec{S} \right)^{-1} \left(\dot{\vec{H}}_j - \vec{\mu} \right) \right]$$

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- Priors
  - $\vec{S}$ Half-normal distribution
  - $\mathbf{R}$ LKJ Distribution
  - $\vec{\mu}$ Exponential distribution

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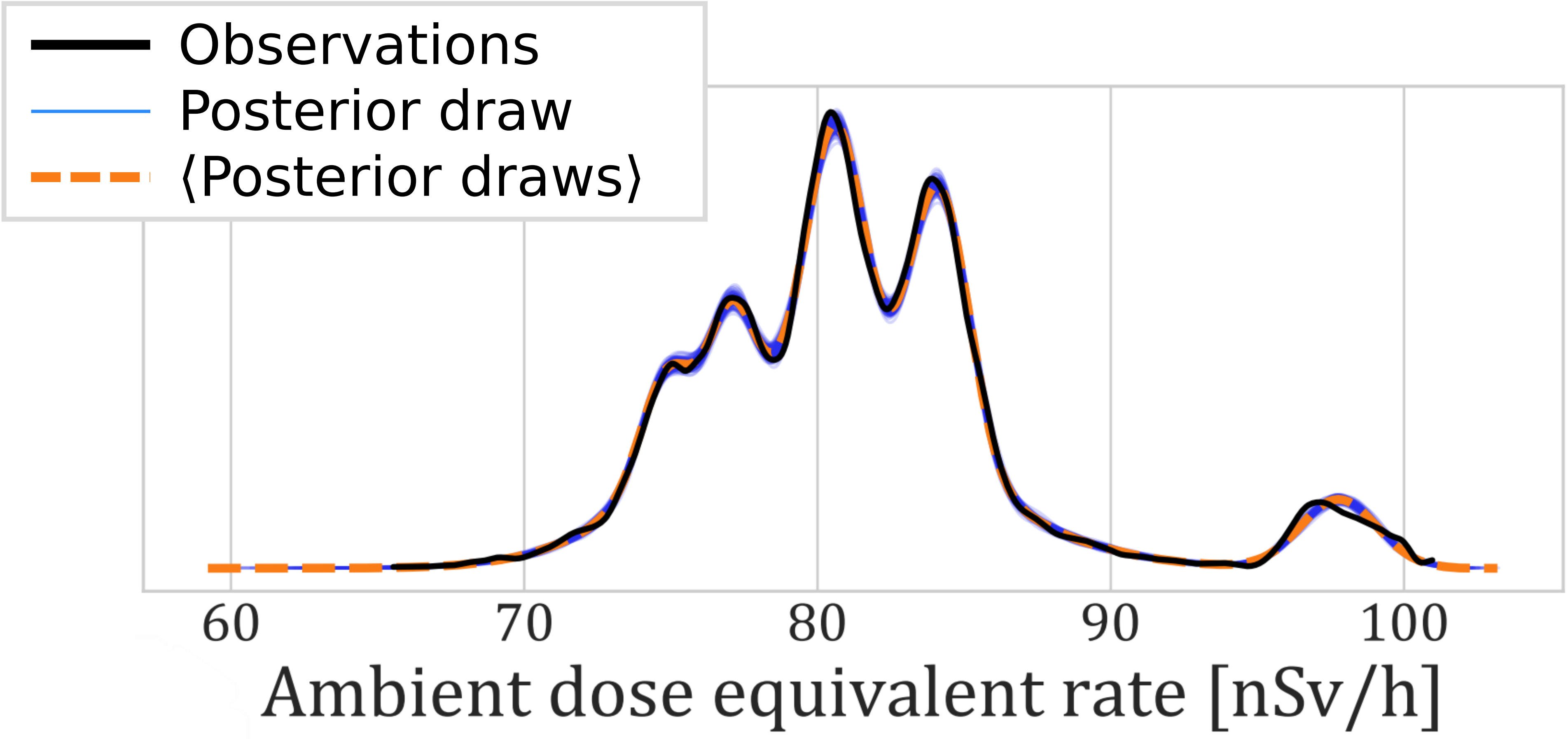
B

Posterior predictive check of calibrated model



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B Predictions using multivariate normal distribution

- Another Bayes's thereom

$$f\left(\vec{H}_{\text{unobserved}} \middle| \vec{H}_{\text{observed}}\right) \propto f\left(\vec{H}_{\text{o}} \middle| \vec{H}_{\text{u}}\right) f\left(\vec{H}_{\text{u}}\right)$$

- If $\vec{H} \sim \mathcal{N}(\vec{\mu}, \mathbf{\Sigma})$:
  - Analytical solution exists!
  - We can calculate $\vec{\mu}_{\text{u}|\text{o}}$ and $\mathbf{\Sigma}_{\text{u}|\text{o}}$

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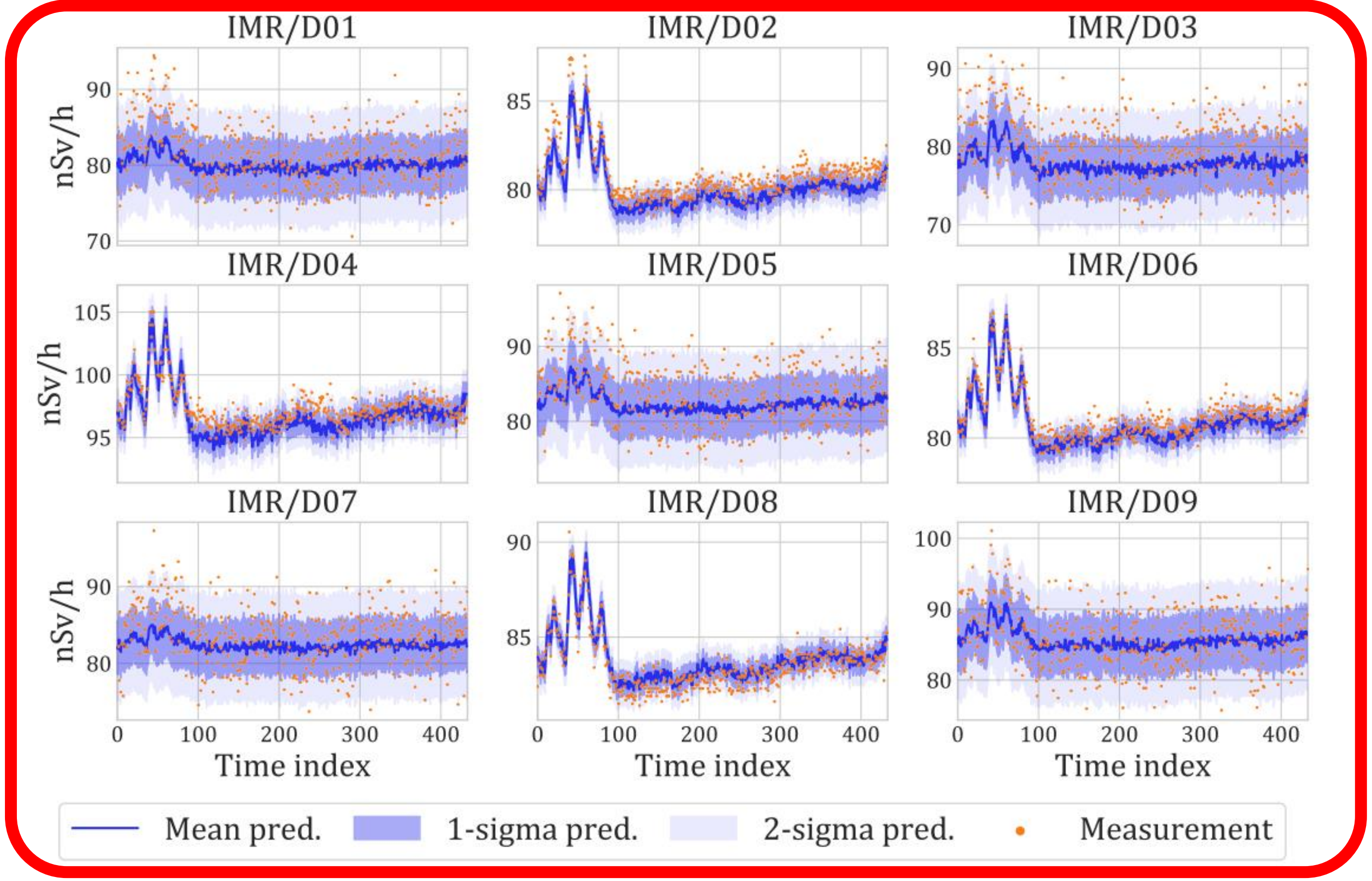
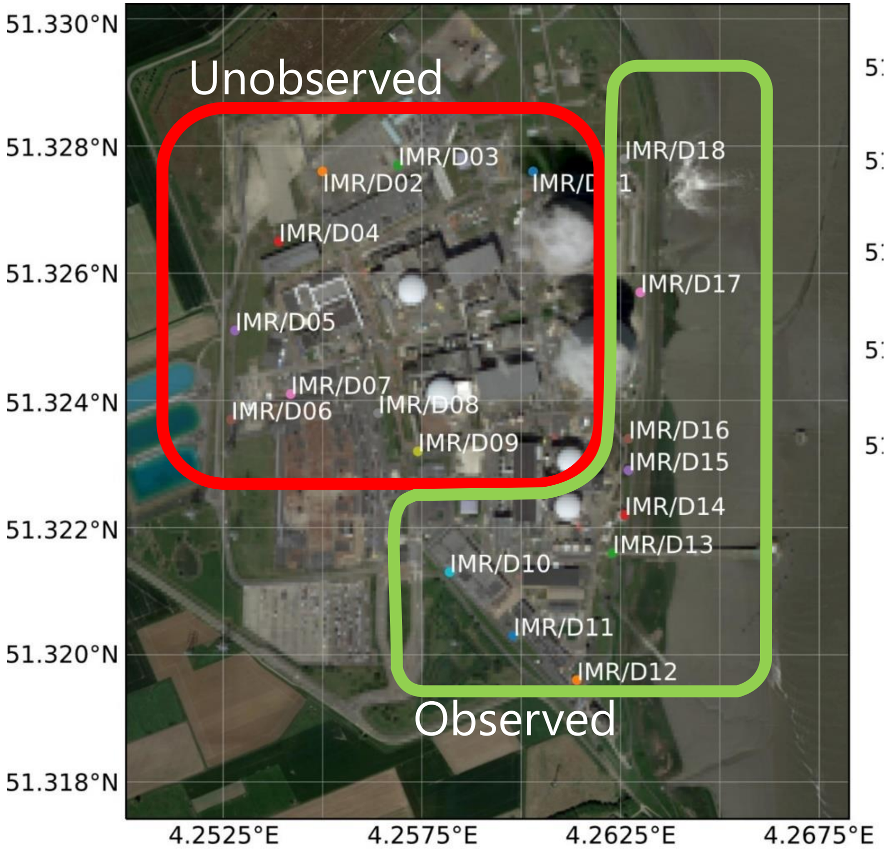
B

Predictive modelling of unobserved detectors with the calibrated Bayesian model





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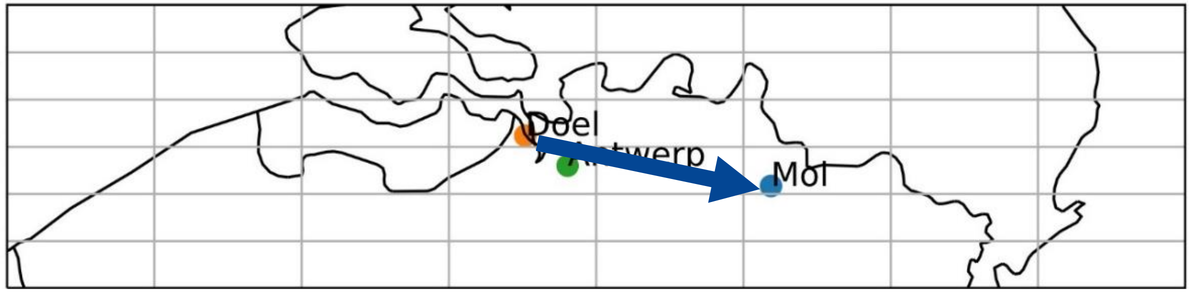
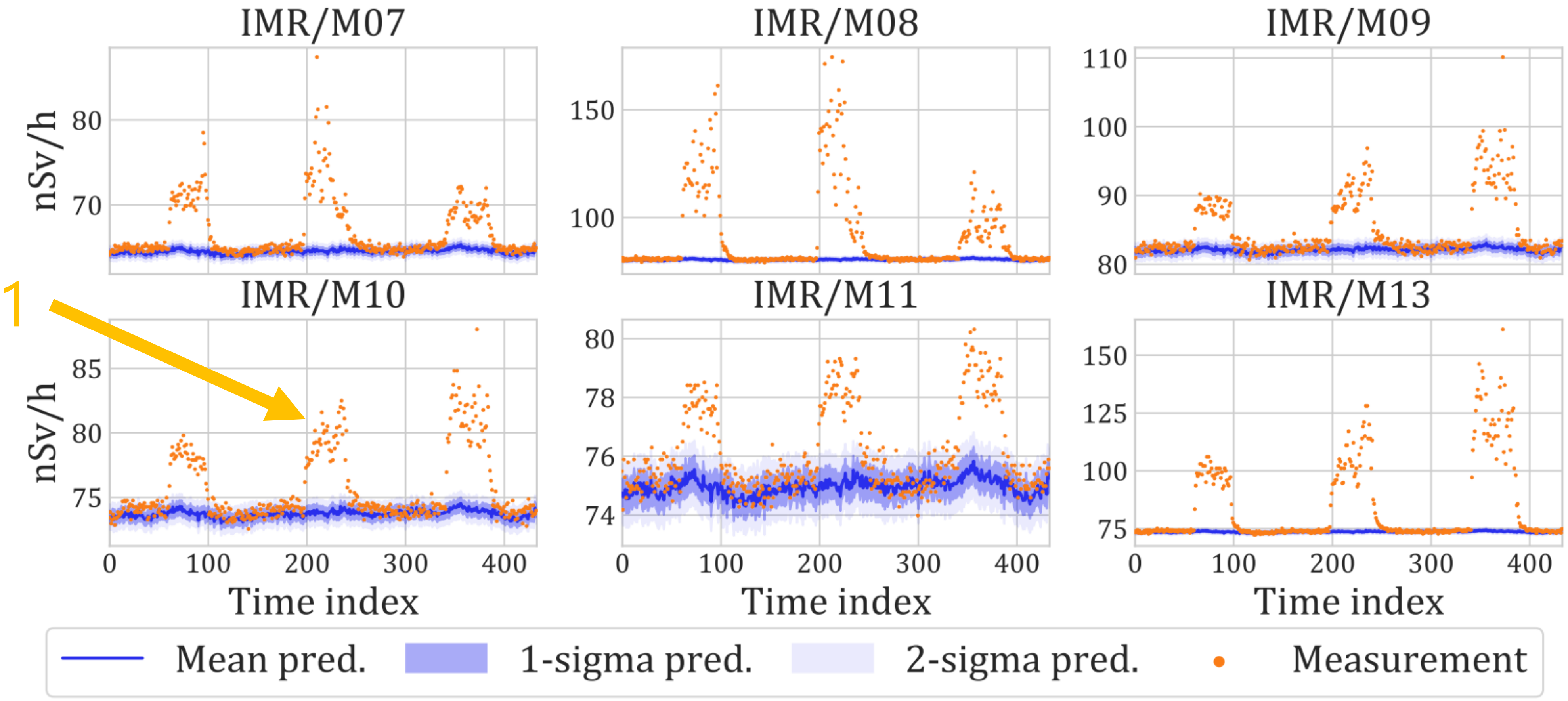
B

Predictions work 60 km apart: very powerful!



BR1



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C Simplified source inversion framework incl. background estimates

- Observations

$$Q_{o,i} = \frac{(\text{Observation by det. } i) - (\text{Background estimate for det. } i)}{\text{ADDER estimate for det. } i}$$

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Simplified source inversion framework incl. background estimates

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C

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- Priors
  - σ half-normal distribution
  - Q_a exponential distribution
  - μ Fixed to ensure $E[Q_{o,i}/Q_a] = 1$

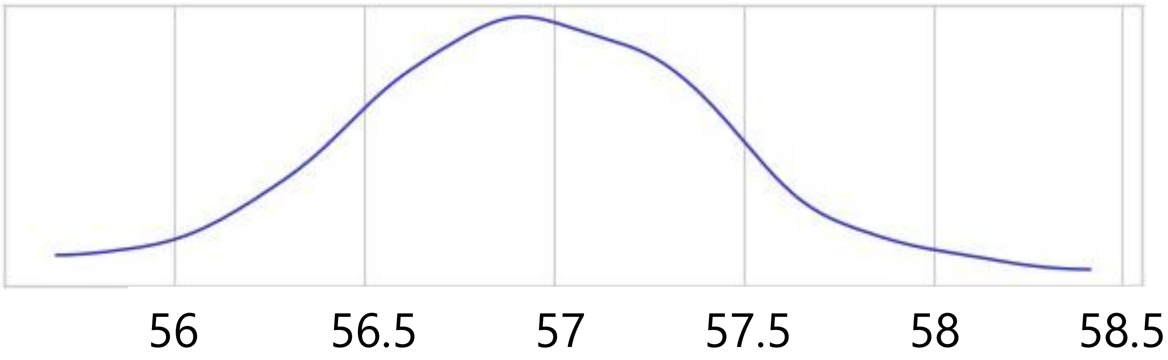
PyTensor

PyMC

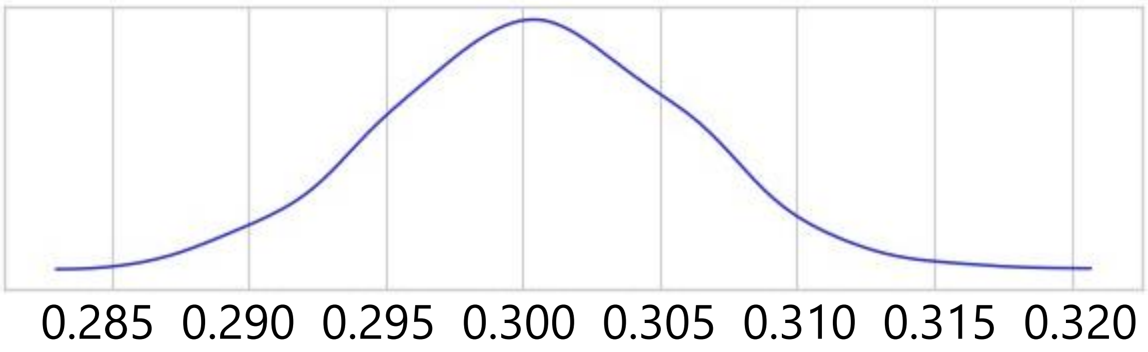
C

Source term estimate and dispersion model error

Source term Q_a (MBq/s)



Model error σ

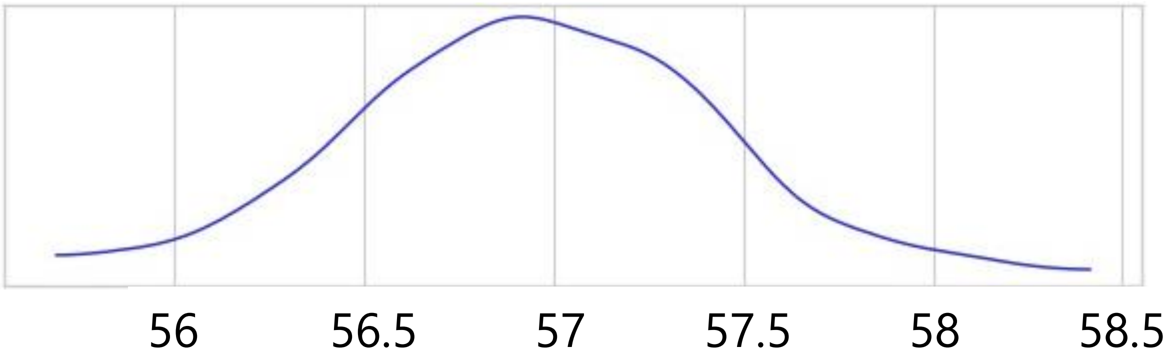


- Most likely source term:
  - $\boldsymbol{Q = 200\ \mathrm{GBq/h}}$
- Model error
  - $\boldsymbol{\sigma = 0.3}$

Source term estimate and dispersion model error

C

Source term Q_a (MBq/s)



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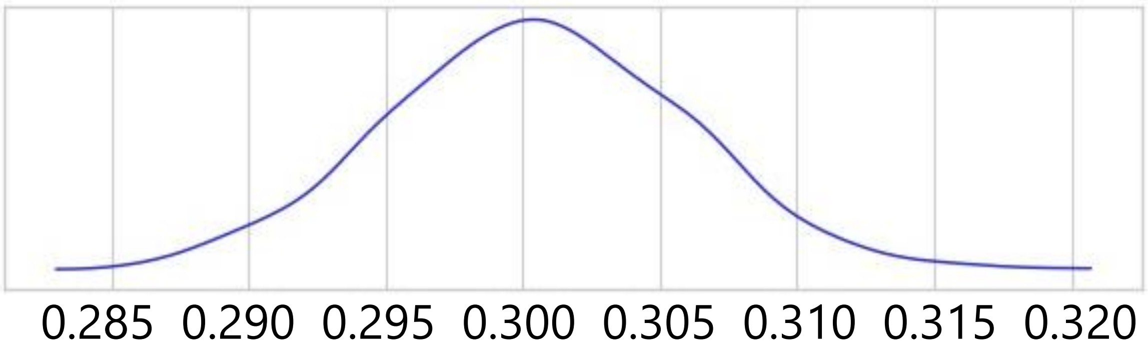

- Most likely source term:
  - $Q = 200\ \mathrm{GBq/h}$
- Model error
  - $\sigma = 0.3$
- In line with previous work on source term estimation at BR1
  - Bijloos et al (2020)
  - Frankemölle et al (2022)

Bijloos, G. et al. *J. Environ. Radioact.* **225**, 106445 (2020)
Frankemölle et al. *HARMO21* (2022)

Conclusions and outlook
Towards a comprehensive framework

- ADDER dispersion model
  - Works in forward (^{75}Se@BR2) and backward mode (^{41}Ar@BR1)
  - Univariate lognormal likelihood works for the ring detectors
    - *Work in progress*: extending to **multivariate** lognormal likelihood to capture downwind correlation of errors

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- Background model
  - Multivariate normal parameterisation for likelihood is very good
  - (Spatial) predictive modelling works well

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- Background model
  - Multivariate normal parameterisation for likelihood is very good
  - (Spatial) predictive modelling works well
- ADDER + Background model:
  - *Work in progress*: Calculate the entire posterior in one go?
  - *Work in progress*: Estimate, e.g., dispersion coefficients?