H22-056:
A methodology to derive Monin-Obukhov universal functions consistent with second order turbulence models and application to dispersion using hybrid moment/PDF methods

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code_saturne

Goal and Objectives

Objectives

- Present methodology to obtain idealised description of SBL flows for inlet boundary condition or idealised carrier flows
  - Coherent with Monin–Obukhov similarity theory
  - Coherent with the turbulence model selected
  - Obtain profiles for first and second order turbulent moments
- Application on pollutant dispersion using hybrid moment/PDF methods
  - Effects of stability
  - Effects of thermal modelling

Effects of the stability on surface-boundary-layer flows

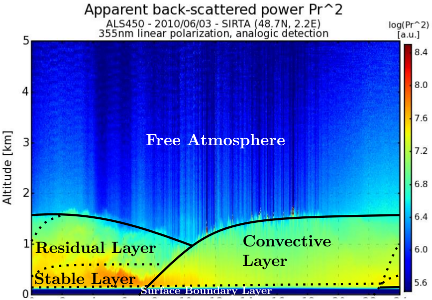
Thermal effects

- Adiabatic dilatation with altitude → cooling.
- Thermal forcing

Unstable case
Enhance vert. turb.

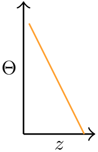
Stable case
Damper vert. turb.

Thermal forcing and atmospheric boundary layer

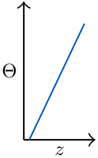


Impact on plume dispersion

Unstable case:



Stable case:



Effects of the stability on surface-boundary-layer flows

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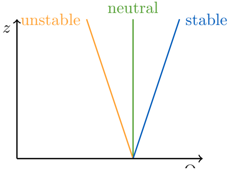
Potential temperature $\Theta = T \left(\frac{P}{P_0}\right)^{-\frac{R}{C_p}}$

- Accounts for adiabatic dilatation
- Is an entropy measure: $s_{rev} = s_{ref} + C_p \ln(\frac{\Theta}{T_{ref}})$
- Enables to simply define stability condition:
  - $\frac{\partial \Theta}{\partial z} > 0$ stable
  - $\frac{\partial \Theta}{\partial z} = 0$ neutral
  - $\frac{\partial \Theta}{\partial z} < 0$ unstable



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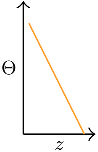

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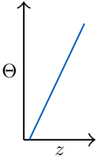
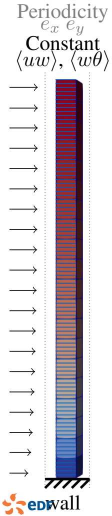


Thermally stratified surface boundary layer



Stratified surface boundary layer characterisation:

- Simple sheared, turbulent uncompressible flow
  - buoyancy → Boussinesq approx.
- Stationary and horizontally uniform flow.
- Neglect Coriolis effects and thermal damping.
- Constant shear stress $\langle uw \rangle = -u_*^2$
- Constant thermal fluxes $\langle w\theta \rangle = -u_* \theta_*$

Valid locally near ground (a few dozen to few hundreds metres)

Monin-Obukhov similarity theory is applicable

- Fields depend on the dimensionless parameter

$$\zeta = \frac{z}{L_{MO}} \quad \text{with} \quad L_{MO} = \frac{u_*^2}{\kappa g \beta_0 \theta_*}$$

- Flow dynamics fully characterised by "universal functions": $\varphi(\zeta)$.

Selection of universal functions

Which universal function to consider?
Numerous proposals φ_m, φ_h for $\langle U \rangle$ and $\langle \Theta \rangle$:

$$\frac{\partial \langle U \rangle}{\partial z} = \frac{u_*}{\kappa z} \varphi_m(\zeta) \quad \text{and} \quad \frac{\partial \langle \Theta \rangle}{\partial z} = \frac{\theta_*}{\kappa z} \varphi_h(\zeta)$$

but triggers different issues:

- No information on turbulent quantities
- Tricky to select one among many
- No respect for theoretical asymptotic behaviour
- Being fitted on experimental setups, they may disagree with numerical simulation → profiles injected are not maintained

Solution kept: to derive universal functions in coherence with the model selected

Basic Laws of Turbulent Mixing in the Ground Layer of Atmosphere 1954, Tr. Akad. Nauk SSSR Geophiz. Inst, A. Monin
Statistical Fluids dynamics Vol. 1 1971, MIT Press A. Monin, A. Yaglom

Derivation of "universal functions" solutions of the algebraic model

Hypothesis considered

- Weak equilibrium assumption on $\langle u_i u_j \rangle$ and $\langle u_i \theta \rangle$

$$\frac{D\langle u_i u_j\rangle}{Dt} - \mathcal{D}_{\langle u_i u_j\rangle} = \frac{\langle u_i u_j\rangle}{k}\left(\frac{Dk}{Dt} - \mathcal{D}_k\right),$$

$$\frac{D\langle u_i \theta\rangle}{Dt} - \mathcal{D}_{\langle u_i \theta\rangle} = \frac{\langle u_i \theta\rangle}{\sqrt{\langle\theta^2\rangle k}}\left(\frac{D\sqrt{\langle\theta^2\rangle k}}{Dt} - \mathcal{D}_{\sqrt{\langle\theta^2\rangle k}}\right),$$

- Equilibrium assumption on k and $\langle\theta^2\rangle$

$$\epsilon = \mathcal{P} + \mathcal{G}$$

$$\epsilon_{\langle\theta^2\rangle} = \mathcal{P}_{\langle\theta^2\rangle}$$

  - + weak equilibrium $\rightarrow$ no diffusion on $\langle u_i u_j\rangle$ and $\langle u_i \theta\rangle$

- Proportionality between thermal and dynamic time scales:

$$\mathcal{C}_{\langle\theta^2\rangle} = \frac{\mathcal{T}_k}{\mathcal{T}_{\langle\theta^2\rangle}} = \frac{k}{\epsilon}\frac{\epsilon_{\langle\theta^2\rangle}}{2\langle\theta^2\rangle}$$

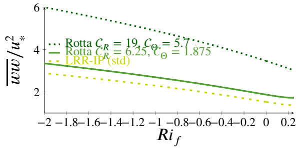
- Linear redistribution model

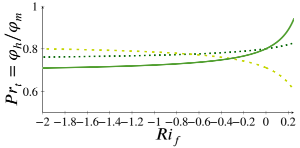
$$\mathcal{R}_{ij} = -\mathcal{C}_R\epsilon\left(\frac{\langle u_i u_j\rangle}{k} - \frac{2}{3}\delta_{ij}\right) - \mathcal{C}_P\left(\mathcal{P}_{ij} - \frac{2}{3}\mathcal{P}\delta_{ij}\right) + C_k k S_{ij} - \mathcal{C}_G\left(\mathcal{G}_{ij} - \frac{2}{3}\mathcal{G}\delta_{ij}\right)$$

$$\mathcal{R}_{\theta i} - \epsilon_{\theta,i} = -\mathcal{C}_{\theta_1}\frac{\epsilon}{k}\langle u_i\theta\rangle - \mathcal{C}_{\theta_2}\mathcal{P}^U_{\theta,i} - \mathcal{C}_{\theta'_2}\mathcal{P}^\Theta_{\theta,i} - \mathcal{C}_{\theta_3}G_{\theta,i}$$

Solution of algebraic model considering $\langle u_i u_j\rangle$ and $\langle u_i\theta\rangle$ equations

- Analytic solution as function of Ri_f with $Ri_f = -\frac{\mathcal{G}}{\mathcal{P}} = \frac{\zeta}{\varphi_m}$





No solution yet for $\varphi_m(\zeta)$ and for $Ri_f(\zeta)$

- Focus on the equation ϵ

with $\epsilon = \frac{u_*^3(1-Ri_f)}{\kappa L_{MO} Ri_f}$ at equilibrium

Resolution of ϵ equation to close the system and derive momentum universal function

Equation on the dissipation rate

$$-\mathcal{D}_\epsilon^S = -\frac{\mathrm{d}\left(\mathcal{C}_{\epsilon S}\frac{k^2}{\epsilon}\frac{\mathrm{d}\epsilon}{\mathrm{d}z}\right)}{\mathrm{d}z} = \frac{\epsilon^2}{k}\left(\mathcal{C}_{\epsilon_1}\frac{\mathcal{P}}{\epsilon} + \mathcal{C}_{\epsilon_2}(\frac{\mathcal{G}}{\epsilon} - 1)\right) \quad (\text{where } \mathcal{C}_{\epsilon_3} = \mathcal{C}_{\epsilon_2})$$

An ordinary differential equation can be obtained injecting algebraic solutions

$$\left(Ri_f'^2 - Ri_f\left(Ri_f'' + 2Ri_f'^2\right) + Ri_f^2 Ri_f''\right)^2 (a_1 - Ri_f a_2)^3 = (1 - Ri_f)^{2+2\delta_{DH}}(1 - a_3 Ri_f)^3 (a_1 - Ri_f a_4)^3$$

- Equation too complex to be resolved analytically.
- Proper theoretical asymptotic results in stable and unstable situations.

Numerical resolution of ϵ based on a 1-D iterative process

Constraints imposed:

- $\epsilon > 0$
- $\mathcal{P} > 0$
- $Ri_f < Ri_f^{cr} \simeq 0.25$
- $\epsilon^{neut.} = \frac{u_*^3}{\kappa z}$

Algebraic model selected:

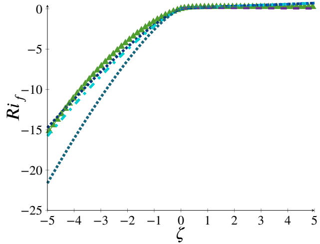
- Boussinesq approximation
- Rotta/SLM on velocity
- Monin/IEM on pot. temp.

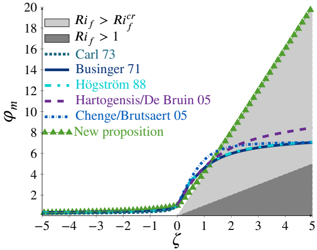
$$\mathrm{d}X_i = \mathrm{d}U_i \mathrm{d}t$$

$$\mathrm{d}U_i = \left(-\frac{1}{\rho_0}\frac{\partial\langle P\rangle}{\partial x_i} - \frac{(U_i - \langle U_i\rangle)}{T_L} + (1 - \beta_0(\Theta - \Theta_0))g_i\right)\mathrm{d}t + \sqrt{\mathcal{C}_0\epsilon}\,\mathrm{d}\mathcal{W}_i \quad \text{with } T_L = \frac{k}{(0.5 + 0.75\mathcal{C}_0)\epsilon} \text{ and } \mathcal{C}_0 = 3.5$$

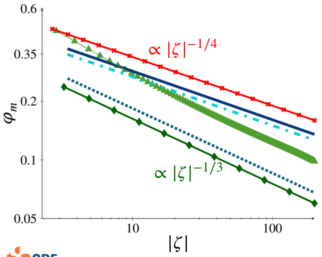
$$\mathrm{d}\Theta = -\frac{\Theta - \langle\Theta\rangle}{T_\Theta}\mathrm{d}t \text{ with } T_\Theta = \frac{k}{\mathcal{C}_\Theta \epsilon} \text{ and } \mathcal{C}_\Theta = 1.875$$

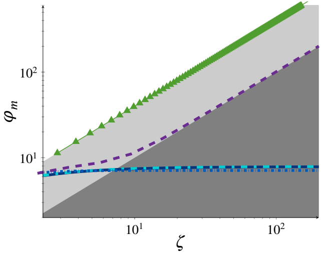
Verification of the universal functions obtained





- Assymptotic behaviours (left convective, right stable):





- Literature Proposals
  - Stable case:
    - reaches zone where $Ri_f > Ri_f^{cr}$
  - Convective limit:
    - poor decrease rate (except Carl 73)
- Proposed methodology
  - Stable case:
    - remains in validity zone
  - Convective case:
    - correct decrease rate ($|\zeta|^{-1/3}$)

Flux-Profile Relationships in the Atmospheric Surface Layer 1971, Journal of the Atmospheric Sciences, J. Businger; J. Wyngaard;and Y. Izumi and E. Bradley.

Profiles of Wind and Temperature from Towers over Homogeneous Terrain 1973, Journal of the Atmospheric Sciences, D. Carl, and T. Tarbell; H. Panofsky.

Non-Dimensional Wind and Temperature Profiles in the Atmospheric Surface Layer: A Re-Evaluation 1981, Topics in Micrometeorology. A Festschrift for Arch Dyer U. Högström.

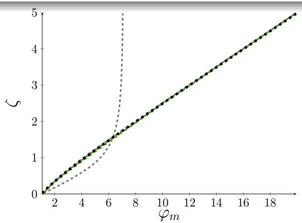
Monin–Obukhov Similarity Functions of the Structure Parameter of Temperature and Turbulent Kinetic Energy Dissipation Rate in the Stable Boundary Layer 2005, Boundary-Layer Meteorology, O. Hartogensis and H. De Bruin.

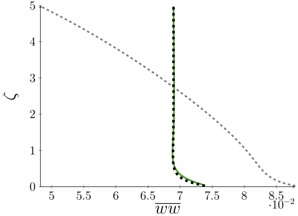
Flux-profile Relationships for Wind Speed and Temperature in the Stable Atmospheric Boundary Layer 2005, Boundary-Layer Meteorology, Y. Chenge and W. Brutsaert

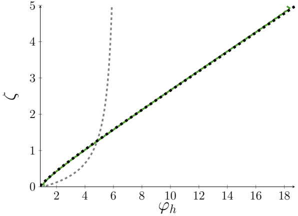
Verification of the universal functions obtained against CFD simulations in stable situation

Verification in a stable situation (L_{MO} = 20 m)

- Initial state: (- - -) (based on k-eps alg. sol and Chenge's φ_m, φ_h)
- Model Coherent solution proposed (■ ■ ■))
- Converged state using code_ saturne (▬▬)







- CFD computation using code_ saturne converges towards the solution proposed
- The method developed enables to properly characterise stable situations (most constraining in the scope of dispersion)

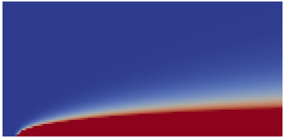
Effects of the stability on the plume shape

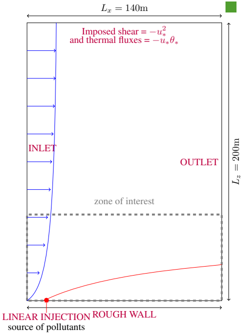
Shape of the plume (see Monin 1959)

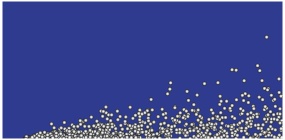
- depends on thermal effects measured by L_{MO}
- is independent on shear velocity u_*

Presentation of the case

- Pollutant emitted at ground
- Only Pollutant followed in PDF methods
- Plume limit: 99% of total mean conc.







Statistical Fluids dynamics Vol. 1 1971, MIT Press A. Monin, A. Yaglom
Smoke propagation in the surface layer of the atmosphere 1959, Advances in Geophysics 6, p331-343, A. Monin

Effects of the stability on the plume shape

Shape of the plume (see Monin 1959)

- depends on thermal effects measured by L_{MO}
- is independent on shear velocity u_*

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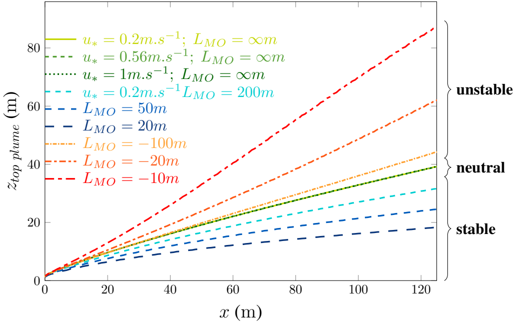
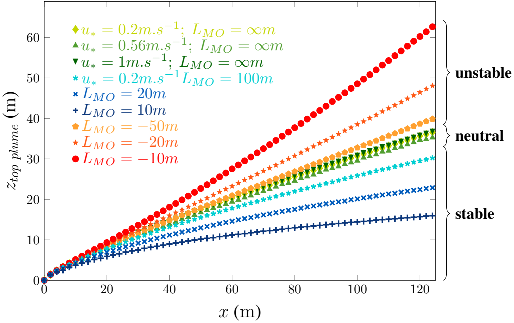
RANS Description



Hybrid Moment PDF Description



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Smoke propagation in the surface layer of the atmosphere 1959, Advances in Geophysics 6, p331-343, A. Monin

Effects of the thermal modelling and stability on the plume shape

Shape of the plume (see Monin 1959)

- depends on thermal effects measured by L_{MO}
- is independent on shear velocity u_*

Thermal model considered

- Finite relaxation time (IEM) $d\Theta = -\frac{(\Theta - \langle\Theta\rangle)}{\mathcal{T}_\Theta}$
- Infinite relaxation time $\Theta = \Theta^{inj}$
- Instantaneous relaxation $\Theta = \langle\Theta\rangle$

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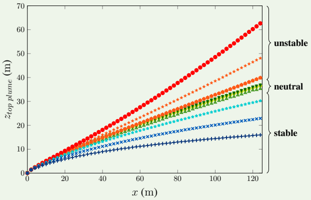
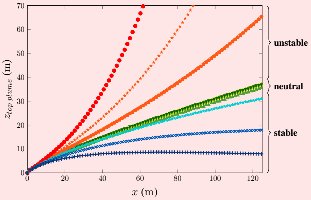
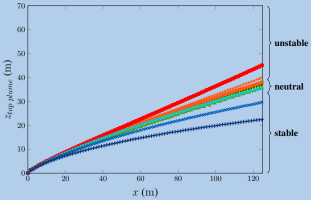
Finite Relaxation time



Infinite Relaxation time



Instantaneous Relaxation time



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Smoke propagation in the surface layer of the atmosphere 1959, Advances in Geophysics 6, p331-343, A. Monin

Influence of thermal modelling on weakly buoyant plume

Presentation of the case

- Stable case: $L_{MO} = 20\,\mathrm{m}$
- Pollutant emitted at 25m
- Weakly Buoyant plume: $\Theta^{inj} = \langle\Theta\rangle(z^{inj}) + 1\mathrm{K}$
- Only the pollutants is followed
- Plume limit: 1%, 50% 99% of total mean conc.

Thermal model considered

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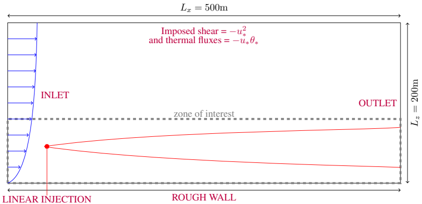

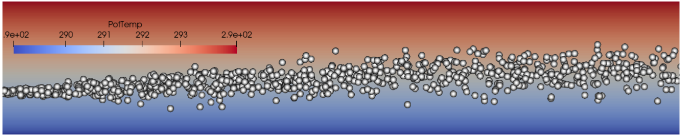

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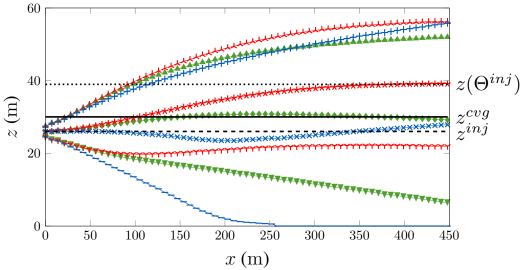

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Impact of thermal modelling on plume dispersion

Thermal relaxation:

- Gouvern the plume rise.
- Dampers plume dispersion.

Development of a methodology to derive model-consistent carrier flows

- Solutions proposed are coherent with Monin-Obukhov theory
- Stable situations well captured by algebraic solutions and iterative process on ϵ
- Further study necessary for unstable cases
- Further study necessary to extend this methodology to the whole atmospheric boundary layer

Stability effect on the pollutant plume shapes retrieved see (Monin 1959)

- No effect of the mean velocity (independent of the friction velocity)
- Dependence of the stability qualitatively assessed

Influence of the modelling of thermal relaxation time

- Affects plume rise
- Affects pollutant dispersion around mean location

Thank you for your attention
Any Question?