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EXTENDED ABSTRACT: A METHODOLOGY TO DERIVE MONIN-OBUKHOV UNIVERSAL FUNCTIONS CONSISTENT WITH SECOND ORDER TURBULENCE MODELS

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Abstract: This work aims to derive universal functions consistent with Monin-Obukhov theory in order to obtain solutions consistent with the modelling of the turbulence considered in the stratified surface boundary layer. Using a Boussinesq assumption on the potential temperature, it is applied to a class of second order turbulence models where the Reynolds tensor, but also the turbulent heat flux and the potential temperature variance are transported. This method is based on the one hand on the resulting algebraic solutions under equilibrium assumptions, and on the other hand on the numerical resolution of the turbulent dissipation rate ε using a one-dimensional iterative process. Providing some constraints on the modelling of the latter, it has been verified that one can then obtain solutions consistent with the Monin-Obukhov theory and the solutions of the computation using the CFD solver code_saturne in a stably stratified surface boundary layer.

Key words: *Algebraic model, Atmospheric stability, Monin-Obukhov theory, Surface boundary layer, Universal functions*

INTRODUCTION

Within the framework of Monin-Obukhov (1954) similarity theory, all mean and turbulent quantities can be written based on so called "universal functions", assuming steady state, homogeneous horizontal flow with constant shear (expressed using the friction velocity u_*) and a constant heat flux (expressed using the friction velocity u_* and temperature θ_*). In absence of measured wind data, universal functions are often used to provide idealised inlet conditions for microscale atmospheric numerical simulations.

Many proposals have been made in the literature to determine such functions for mean velocity and mean potential temperature. However, the selection of a particular proposal can be tricky and not fully satisfactory for several reasons. First, on theoretical grounds, many proposals do not respect the very stable or convective asymptotic behaviours derived from the Monin-Obukhov similarity theory. Furthermore, they do provide information only on the first order moments and are therefore not sufficient to characterise a flow as no information on the turbulent quantities is provided. Finally, these functions are experimentally fitted and may not match with the steady state solution of the turbulence model considered in numerical simulations under constant shear and heat flux assumptions. This triggers the impossibility of maintaining the profiles imposed at inlet in conditions they must maintain. This issue is particularly pronounced in stably stratified situations.

Moreover, the lack of information for the turbulent quantities can lead to different closure choices depending on the user. This also introduces a new source of variability in the results. In this context, the work presented here aims at harmonising the estimation of universal functions by providing a generic method to derive universal functions in coherence with the chosen modelling.

METHODOLOGY FOLLOWED

Derivation of algebraic solutions

To obtain a model-consistent description of surface boundary layer (SBL) flows, we aim to estimate the solutions of the numerical solver to use them as universal functions. To this end, the algebraic solutions consistent with the selected model are derived for a stationary horizontally stratified SBL flow. First a Boussinesq approximation on the potential temperature is made to deal with buoyancy effects. As we consider high Reynolds number flows, the effects of molecular viscosity and diffusivity are neglected, and the dissipation rate of turbulent kinetic energy (TKE) is assumed to be isotropic. Furthermore, in accordance with the Kolmogorov theory, the dynamic and the thermal turbulent relaxation times are assumed proportional. Additionally, the pressure-strain correlation and scrambling terms are modelled as a linear combination of the terms appearing respectively in the Reynolds tensor and heat flux equations. Finally, we assume a weak equilibrium as well as an equilibrium between the production and dissipation terms for the turbulent kinetic energy and the potential temperature variance.

These latter assumptions enable to obtain local equations for the Reynolds tensor, the heat flux vector, and the potential temperature. Injecting all these assumptions into the set of equations considered, we can then derive algebraic solutions for all second-order Reynolds averaged quantities (Reynolds tensor, turbulent heat flux vector, and temperature variance) as functions of a single parameter: the flux Richardson number Ri_f. The latter corresponds to the opposite of the ratio between the production or destruction of TKE by thermal effects G and the production of TKE by shear P. In the flow under consideration, it is related to the momentum universal function ($\varphi_m = \frac{\kappa\, z}{u_*} \frac{\partial\, \bar{U}}{\partial\, z}$ where z is the altitude, $\kappa = 0.42$ the Von Kármán constant and $\bar{U}$ the mean velocity), as follows:

$$Ri_f = -\frac{G}{P} = \frac{\zeta}{\varphi_m}, \qquad (1)$$

with ζ the dimensionless parameter characterising, according to Monin-Obukhov similarity theory, the overall dynamic and thermal processes in the thermally stratified SBL. It is defined as the ratio between the altitude z and the Monin-Obukhov length scale L_{MO} which is the ratio of the shear and buoyant effects.

This step enables to provide universal functions for the mean potential temperature gradient ($\varphi_h = \frac{\kappa\, z}{\theta_*} \frac{\partial\, \bar{\Theta}}{\partial\, z}$ with $\bar{\Theta}$ the mean potential temperature) and second-order moments. Moreover, all these universal functions depend solely on the universal function associated with the mean velocity which has yet to be determined.

Numerical resolution of the dissipation rate equations

At this stage the system considered is not closed as no information about the flux Richardson number and then about the momentum universal functions is provided. To obtain it, we will focus on the TKE dissipation rate ε, which, when the TKE production-dissipation equilibrium is reached, is related to the flux Richardson number as:

$$\varepsilon = G\left(\frac{Ri_f - 1}{Ri_f}\right). \qquad (2)$$

We consider then the transport equation of ε, written as follows:

$$-D_\varepsilon = -\frac{\mathrm{d}\left(C_\varepsilon \frac{k^2}{\varepsilon}\frac{\mathrm{d}\,\varepsilon}{\mathrm{d}\,z}\right)}{\mathrm{d}\,z} = \frac{\varepsilon}{k}\left(C_{\varepsilon 1}\, P + C_{\varepsilon 3}\, G - C_{\varepsilon 2}\varepsilon\right), \qquad (3)$$

where a simple gradient diffusion hypothesis is applied for the diffusion term, with $C_{\varepsilon 1} = 1.44$ and $C_{\varepsilon 2} = 1.92$ taken from Launder and Spalding (1972), and $C_{\varepsilon 2} = C_{\varepsilon 3}$ taken to enable the attainment of the desired asymptotic behaviour in the convective situations. Injecting the algebraic solution for the TKE into this equation, we can determine an ordinary differential equation of which Ri_f is the solution. Unfortunately, the latter equation is too complex to be resolved analytically. Yet it is employed to ensure that the asymptotic behaviours obtained with this method are in accordance with Monin-Obukhov similarity theory.

It has then been chosen to resolve the dissipation rate of the TKE, numerically using a one-dimensional iterative process whereby the SBL is spatially discretised. During this resolution, certain constraints are imposed. These include the requirement to have, at all points and at all times, the positivity of the dissipation rate and production by shear but also to have an upper limit for the Ri_f bounded by its critical value $Ri_f^{cr} \approx 0.25$. Once these constraints have been satisfied, the results are corrected in order to obtain the proper results in the neutral case.

VERIFICATION

Consistence with Monin-Obukhov similarity theory

Firstly, the results obtained with the new methodology are compared with other commonly used proposals from the literature in Figure 1. In the stable case (right-hand side), it can be observed that the solution obtained with the new methodology converges well towards the limit $Ri_f = Ri_f^{cr}$, whereas all profiles from the literature plotted converge towards a limit $Ri_f > Ri_f^{cr}$.

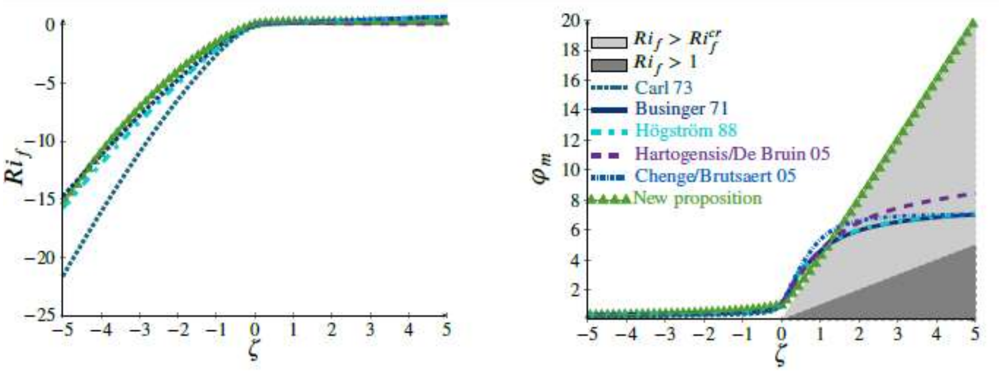

Figure 1. Profiles of the flux Richardson number and momentum universal function derived with the proposed methodology for Rotta-Monin modelling compared to literature proposals and theoretical behaviour.

One can focus on the asymptotic behaviour by taking greater value of ζ, In Figure 2. In the stable limit (right-hand side), the proposal of Hartogensis and Debruin (2005) converges towards an equilibrium between production by shear and destruction by thermal effects $P = G$ reached when $Ri_f = 1$. In this case, the dissipation rate tends towards zero whereas it tends towards a negative value for the other literature profiles. The results obtained with the proposed methodology tend towards the correct asymptotes. In the convective limits, this proposal and the one of Carl (1973*)* converge with the power -1/3, which is theoretically expected (see Monin and Yaglom (1971)). The other proposals converge too slowly with a power -1/4. Therefore, it can be concluded that the method proposed enables to reach solutions consistent with Monin-Obukhov asymptotic behaviour.

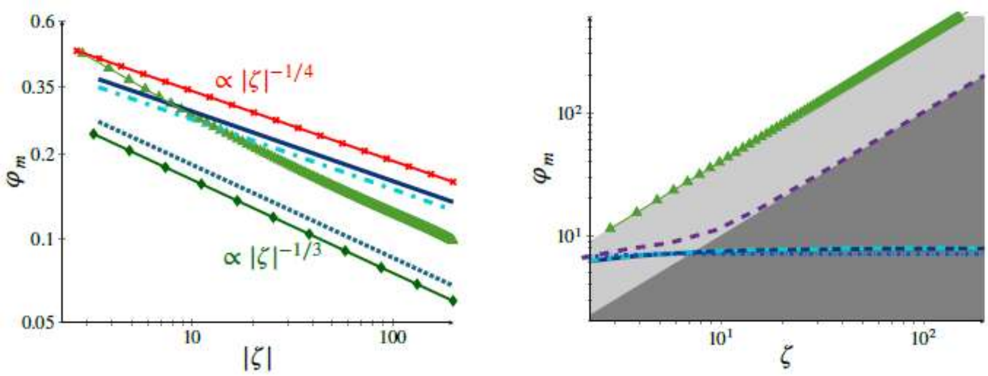

Figure 2. Asymptotic behaviour of the momentum universal function derived with the proposed methodology for Rotta-Monin modelling compared to commonly employed literature proposals and theoretical asymptotes.

Verification in a stable case with numerical simulations of code_saturne

To ensure that the methodology proposed is coherent not only with the Monin-Obukhov theory but also with our choice of modelling, the results obtained in this manner are compared to numerical simulations carried out using code_saturne (see Archambeau (2004)). In order to achieve this, a 100 m height stable SBL is considered with a Monin-Obukhov length scale L_{MO} of 20 m. This situation is the most challenging in terms of maintaining injected profiles and in the scope of atmospheric pollutant dispersion. The corresponding results are plotted on Figure 3. The numerical simulation is initialised (grey lines) based on universal profiles from Chenge and Brutsaert (2005) and algebraic solutions associated with the standard k-ε model, i.e., not consistent with the Rotta-Monin model considered. The code_saturne simulation is carried out until reaching a steady state (in green), which notably different from the literature initial state but fits almost perfectly the results obtained with the proposed methodology (black dotted lines).

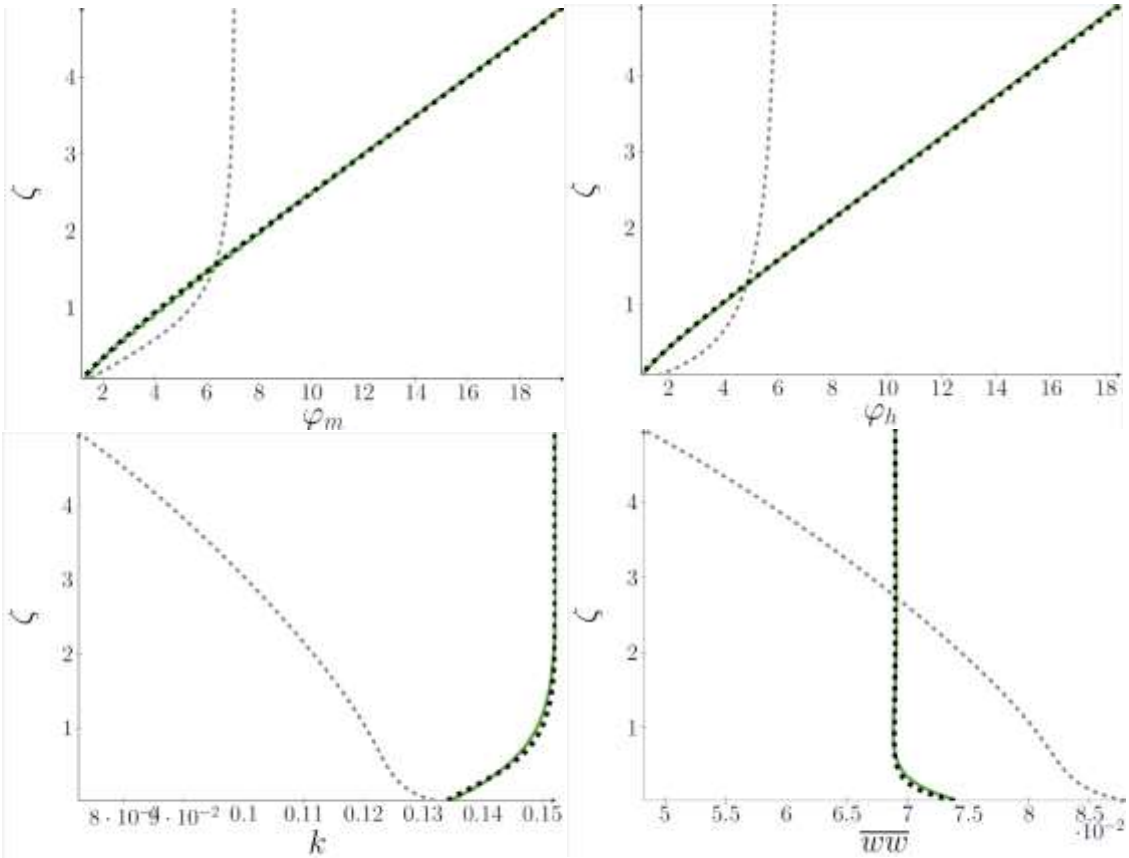

Figure 3. Vertical profiles of a converged stably stratified SBL using code_saturne (green lines). These results are compared to those obtained with the proposed methodology (black dotted lines) and the initial state imposed based on literature profiles and algebraic solutions from another model.

It can be concluded that the proposed methodology provides a highly accurate model-consistent description of the stably stratified SBL. In this case, the equilibrium assumptions made correctly characterised the flow as the diffusion terms on the second-order moments play indeed little role.

CONCLUSIONS AND PERSPECTIVES.

A methodology enabling the derivation of model-consistent description of the surface boundary layer has been proposed. On the one hand it is based on model algebraic solutions obtained through equilibrium assumptions. These solutions can then be obtained as functions of the flux Richardson number. On the other hand, an iterative equation of the dissipation rate equation is carried out. The resolution of this non-local equation provides a notion of distance to the ground which is useful for determining a model-consistent momentum universal function and flux Richardson number. The solutions obtained in this manner are coherent with Monin-Obukhov theoretical behaviour. In the stable case, where the equilibrium assumption proposed represents well the physics involved, this methodology enables to obtain a proper description of the flow. Further investigation is required in the case of convective situations where the production-dissipation equilibrium hypotheses no longer hold.

REFERENCES

F. Archambeau, N. Méchitoua, and M. Sakiz., 2004: Code Saturne: A finite volume code for the computation of turbulent incompressible flows-Industrial applications. *International Journal on Finite Volumes*, **1(1)**.

J. A. Businger, J. C. Wyngaard, Y. Izumi, and E. F. Bradley, 1971: Flux-Profile Relationships in the Atmospheric Surface Layer. *Journal of the Atmospheric Sciences*, **28(2)**, 181–189.

D. M. Carl, T. C. Tarbell, and H. A. Panofsky, 1973 : Profiles of Wind and Temperature from Towers over Homogeneous Terrain. *Journal of the Atmospheric Sciences*, **30(5)**, 788–794.

Y. Chenge and W. Brutsaert 2005. Flux-profile Relationships for Wind Speed and Temperature in the Stable Atmospheric Boundary Layer. *Boundary-Layer Meteorology*, **114(3)**, 519–538.

O. K. Hartogensis and H. A. R. De Bruin, 2005: Monin–Obukhov Similarity Functions of the Structure Parameter of Temperature and Turbulent Kinetic Energy Dissipation Rate in the Stable Boundary Layer. *Boundary-Layer Meteorology*, **116(2)**, 253–276..

U. L. F. Högström,1998: Non-Dimensional Wind and Temperature Profiles in the Atmospheric Surface Layer: A Re-Evaluation. *In B. B. Hicks, editor, Topics in Micrometeorology*, 55-75.

B. E. Launder, D. B. Spalding, 1972: Lectures in mathematical models of turbulence. *New York, Academic Press*.

A. Monin and A. M. Obukhov, 1954: Basic laws of turbulent mixing in the ground layer of atmosphere. *Tr. Akad. Nauk SSSR Geophiz. Inst*,.

A. Monin and A. Yaglom 1971: Statistical Fluid Mechanics, Vol. I, Mechanics of Turbulence. *MIT Press, Cambridge*