A non-dimensional urban dispersion model and spurious self-correlation estimate

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Summary

A Gaussian plume urban dispersion model is presented, where the horizontal and vertical diffusion coefficients are determined according to the theories of Taylor (1921) and Hunt and Weber (1979) respectively. The model is validated with concentration measurements from field experiments conducted in Oklahoma City, Salt Lake City, London, and St. Louis. A statistical analysis of robustness is conducted on the data to determine the spurious self-correlation between the non-dimensional variables.

Motivation

The extent of stratification in urban areas and its effects on dispersion are not well understood. The mechanical generation of turbulence and the release of thermal energy accumulated during the day contribute to weakening the stability of the flow.

Model

A reflected Gaussian plume model for ground-level releases is derived under the assumptions of daytime neutral stability and nighttime near-neutral stability:

$$C = \frac{Q}{\pi U \sigma_y \sigma_z} \exp\left(-\frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}\right) \quad (1)$$

$$\sigma_y^2 = \sigma_{yo}^2 + 2\sigma_v^2 T_y^2 \left[\frac{t}{T_y} + \exp\left(-\frac{t}{T_y}\right) - 1\right] \quad (2)$$

$$\sigma_z^2 = \sigma_{zo}^2 + \frac{b^2\sigma_w^2 t^2}{1 + b^2\sigma_w^2 t^2 \pi/(2L_z^2)} \quad (3)$$

Eq. (2) is written according to Taylor (1921). Eq. (3) is modified with respect to Hunt and Weber (1979) to include the boundary layer height at L_z. The empirical constant b depends on the atmospheric stability.

Experiments

Table: Estimates of turbulence and flow characteristics in the experiments.

Experiment	Stability	$\overline{U}$ (ms^{-1})	L_y (m)	L_z (m)	σ_v (ms^{-1})	σ_w (ms^{-1})	T_y (s)	T_z (s)
Salt Lake City	nighttime	0.49	1000	200	0.25	0.16	4082	1237
	daytime	1.03	2000	800	0.52	0.34	3883	2354
Oklahoma City	nighttime	2.08	1000	200	0.99	0.68	1010	294
	daytime	2.13	2000	800	1.09	0.70	1835	1143
St. Louis	nighttime	2.72	1000	200	0.45	0.30	2208	669
	daytime	2.79	2000	800	1.76	1.20	1137	665
London	daytime	3.00	2000	800	1.08	0.72	1845	1118

Results

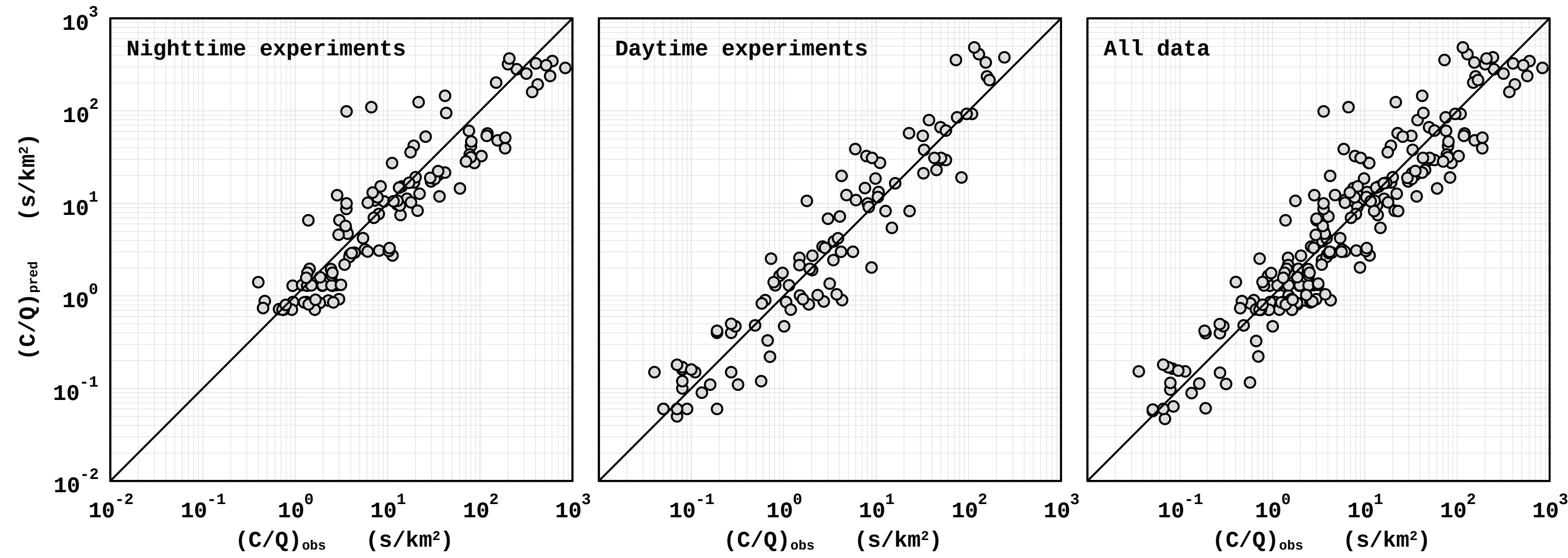

Table: Statistical measures of error between modeled and observed scaled concentration C/Q: correlation (Corr), fractional bias (FB), normalized mean square error (NMSE), geometric variance (VG), and percentage of predictions falling within a factor two of the observations (Fac2).

	Corr	FB	NMSE	VG	Fac2
Nighttime	0.83	0.34	1.62	1.92	63.64
Daytime	0.84	0.50	2.19	1.82	64.71
All data	0.73	0.07	1.78	1.87	64.13

Non-dimensional representation

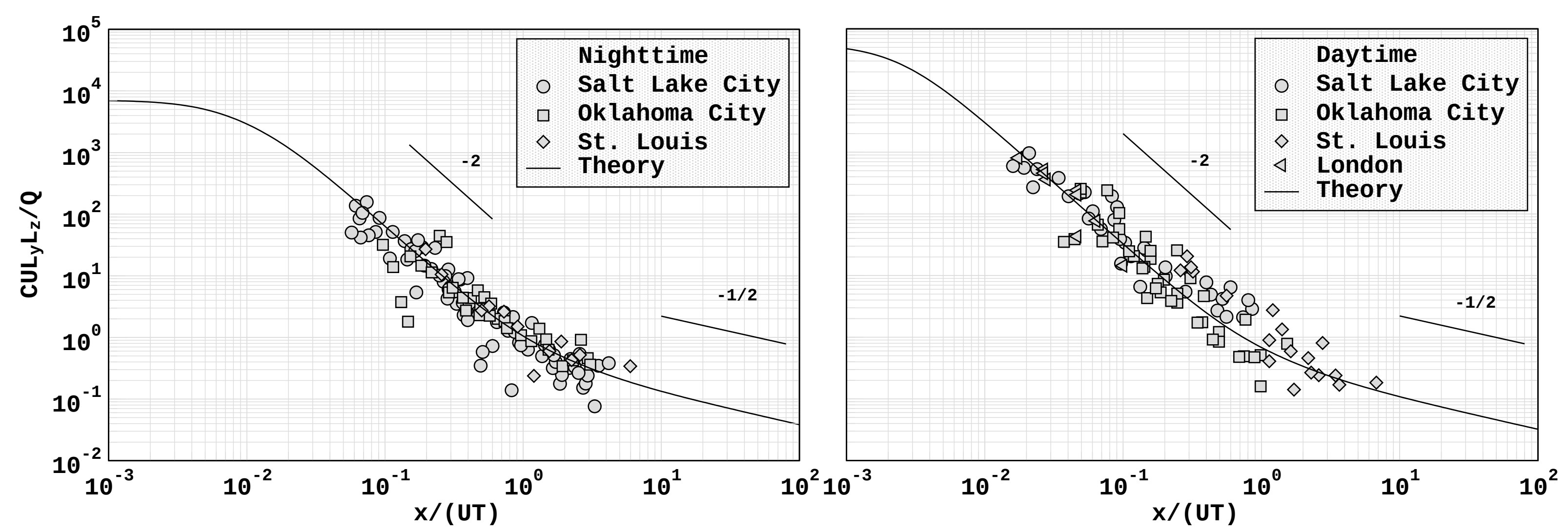

Table: Statistical measures of error between modeled and observed non-dimensional concentrations CUL_yL_z/Q.

	Corr	FB	NMSE	VG	Fac2
Nighttime	0.83	0.29	1.41	1.65	79.34
Daytime	0.91	0.53	1.58	2.13	53.92

Asymptotic behavior

$$C = \left(\frac{Q}{L_y L_z}\frac{UT_yT_z}{\pi b}\right) x^{-2} \quad \text{for } x \ll UT_z \quad (4)$$

$$C = \left(\frac{Q}{L_y L_z}\sqrt{\frac{T_y}{4\pi U}}\right) x^{-1/2} \quad \text{for } x \gg UT_y \quad (5)$$

Spurious correlation analysis

Since the non-dimensional groups CUL_yL_z/Q and $x/(UT)$ both include the variable U, there is in principle the possibility that the observed relationship between the groups is partially spurious. Statistical robustness tests show that the amount of spurious correlation introduced by the common variable U is negligible.

Table: Statistical measures characterizing the linear regressions $\log(CUL_yL_z/Q)$ vs $\log(x/UT)$ and $\log(CL_yL_z/Q)$ vs $\log(x/UT)$: coefficient of determination R^2, regression coefficient (or slope) β, its standard error δ, t-test statistic of the null hypothesis $\beta = 0$, probability p of obtaining a larger t assuming $\beta = 0$, and 95% confidence intervals for both daytime and nighttime cases.

			R^2	β	δ	t	$p > \|t\|$	95% Conf. int.	
$\log(CUL_yL_z/Q)$	vs	$\log(x/UT)$	0.89	−1.57	0.06	−28.29	0.00	[−1.68, −1.46]	day
$\log(CL_yL_z/Q)$	vs	$\log(x/T)$	0.90	−1.61	0.05	−30.42	0.00	[−1.71, −1.50]	
$\log(CUL_yL_z/Q)$	vs	$\log(x/UT)$	0.85	−1.46	0.06	−25.89	0.00	[−1.57, −1.35]	night
$\log(CL_yL_z/Q)$	vs	$\log(x/T)$	0.84	−1.32	0.05	−24.96	0.00	[−1.42, −1.21]	

Conclusion

- Simple, analytical model
- Accounts for the differences between nighttime and daytime atmosphere
- Accounts for the different trends between near and far field
- Identification of time and length scales governing urban dispersion
- Stratification in urban areas is weak but its effects on dispersion are not negligible

References

Hunt, J. C. R., Weber, A. H., 1979. A Lagrangian statistical analysis of diffusion from a ground-level source in a turbulent boundary layer. Quart. J. Roy. Meteor. Soc. 105 (444), 423–443.

Taylor, G. I., 1921. Diffusion by continuous movements. Proc. Lond. Math. Soc. 20, 196–211.

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