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A NOVEL WIND PROFILE FORMULATION FOR NEUTRAL CONDITIONS IN URBAN ENVIRONMENT

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Abstract: One of the main problems to solve in micrometeorological studies concerns the theoretical evaluation of the wind profile in an urban environment. In such a case, in fact, the classical similarity theory gives in general invalid results. Based on this evidence, observations of wind profiles from ground level up to 200 m were collected inside an urban park located in the city of Rome (Italy), by means of a Doppler SODAR during the years 2005 and 2006. Measurements taken by an ultrasonic anemometer and by a conventional meteorological station located at ground level were also used for the study. A new formulation for the vertical profile of the wind velocity in the case of neutral condition is proposed. The results show that the new formulation, which is based on the concept of roughness length varying with the height, performs better than the classical theoretical law based on a constant value of the roughness length.

Key words: Wind profile, Neutral condition, Similarity theory, Urban site, Roughness length

INTRODUCTION

One of the main problems associated with the micrometeorological studies is the theoretical evaluation of the wind profile in urban environment, where the classical similarity theory is generally not appropriate. As is well-known, the presence of the urban features makes the description of the flow field very difficult. For example, the mean horizontal velocity U as a function of the height Z is not longer described by the classical logarithmic shape (Britter R.E. and S.R. Hanna, 2003). This problem is usually overcome extending the relationship valid for flat terrain and for statically neutral condition in the presence of roughness elements, by defining a suitable displacement height d_0 :

$$U(Z) = \frac{U^*}{k} \ln \left(\frac{Z - d_0}{Z_0} \right) \quad (1)$$

where Z_0 is the roughness length, U^* the friction velocity, $k=0.4$ the von Karman constant and $U=0$ for $Z=Z_0+d_0$. The inconvenience of equation (1) is that d_0 and Z_0 are not easy to obtain, in particular in the case of untypical surfaces. Gryning, S.E. *et al.* (2007) proposed a new relationship for the velocity profile, which can be also applied to urban environment. As stated by those authors, such relation, based on the knowledge of the boundary layer depth and other parameters, is very complicated and difficult to use.

In light of that, an investigation is therefore performed with the aim of finding an alternative relationship for the wind profile valid in the case of urban environment. The analysis is performed using a set of meteorological data taken from a site located within the city of Rome, Italy. In particular wind and temperature profiles, taken from ground level up to 200 m above the ground level (agl) during 2005-2006 years by means of a SODAR/RASS system, were used for the analysis. The vertical resolution of the measurements of the latter system is 20 m, while the maximum height is nearly 400 m agl. The first range gate is centered at 40 m agl. The instantaneous vertical profiles (one every 2.5 seconds) are averaged over 10 min, which resulted in a total number of nearly 50000 profiles suitable for the analysis. In addition, a station located in the same site operated and recorded routine meteorological data. In particular, an ultrasonic anemometer (sample rate of 10 Hz) was used to calculate the friction velocity every 10 min at $Z=10$ m agl, while by means of a couple of thermistors positioned at 2 m and 10 m agl, the vertical temperature gradient was calculated. From the latter, information about the static stability of the flow was extracted.

Data were selected for static neutral condition and only those profiles corresponding to wind velocity increasing logarithmically with height were selected for the analysis. It was found that the classical roughness length based on the concept of height where the wind is zero has to be modified for an urban area. As expected, in most cases the classical similarity theory under neutral conditions failed to reproduce the observed wind profile. Thus, we propose an alternative approach that, in order to calculate a novel formulation for $U(Z)$, introduces a new definition of roughness length. The new roughness length is a suitable variable related to the local roughness characterizing the urban building height of Rome.

To test the applicability of the new formulation, its results are compared with those obtained by means of the classical similarity theory. Three values of the roughness lengths were adopted in the classical formula, related to the building heights characterizing the city of Rome. To summarize the results and to detect different abilities of both the classical and the new formulation over different fractions of the wind profile, average differences were calculated separately for the upper (100 m-200 m) and for the lower part of it (10-100 m).

THE ROUGHNESS LENGTH AS A PARAMETER VARIABLE WITH HEIGHT

The classical roughness length Z_0 is defined as the height in which the mean wind velocity becomes zero. However, in the case of complex terrain it is necessary to specify some concepts. Generally, Z_0 is determined according to the available data. The simpler criteria are applied when meteorological data are not available. For example, Z_0 may be calculated by means of the classification founded on the nature of the terrain (Mc Rae, G.J. *et al.*, 1982); a second empirical method is based on the value of the average height of the buildings representative of the investigated area (Zannetti, P., 1990).

The use of methods of calculation of the roughness length Z_0 based on meteorological data is much more complicated. Estimation of Z_0 must be carried out using data measured above a reference altitude, which must show certain criteria related to the average height of the obstacles characterizing the measurement site (Wieringa, J., 1993).

For statically neutral conditions, the average speed at a reference level, namely Z_{rif} , can be expressed through equation (1). Omitting the displacement height, given wind speed observations at two heights, equation (1) can be solved for the two unknowns U^* and Z_0 . By using the same relationship for the whole vertical profile and treating U^* as a constant parameter, a vertical profile for Z_0 may be obtained by equation (1), viz.:

$$Z_0(Z) = \frac{Z_{rif}}{\exp\left[\frac{U(Z_{rif}) \cdot k}{U^*}\right]} \tag{2}$$

Obviously, such a profile has a physical meaning only if U^* does not vary too much with height. This assumption is not always verified. Zilitinkevich, S.S. and N. Esau (2005) proposed a method for calculating the variation of U^* along the vertical direction based on the boundary layer height. That analysis, applied for the present case, shows that U^* normally decreases with height less than 30% in the lowest two hundred meters, so that equation (2) can be considered valid, with a sufficient kind of accuracy, within the boundary layer thickness analyzed in this work.

Table 1 lists both the average and the root mean square (rms) of Z_0 as a function of the height inferred from equation (2) assuming, for each height level, the value of U^* measured at $Z=10$ m. The results show that the roughness length decreases much with height, showing a minimum $Z_0=0.673$ m at $Z=200$ m agl. The vertical profile of Z_0 can be represented by a simple power law (Figure 1):

$$Z_0(Z) = \alpha_{Z_0} \cdot Z^{\gamma_{Z_0}} \tag{3}$$

where $\alpha_{Z_0}=13.37$ and $\gamma_{Z_0}=-0.54$. The average values are well reproduced ($R^2=0.842$). The behavior of Z_0 seems to have a physical meaning in that it decreases with height tending towards a constant value, which is quite similar to that obtained by inferring canonical roughness length based on the terrain type of the city of Rome. In addition, the decreasing of Z_0 is coherent with the presence of internal boundary layers within internal boundary layers, typical of flow over patchwork land-use patterns and urban boundary layer too.

Table 1. Mean and rms of Z_0 calculated with equation (2)

Z (m)	10	40	60	80	100	120	140	160	180	200
$\langle Z_0 \rangle$	2.841	3.061	1.624	1.289	1.072	1.012	0.917	0.789	0.761	0.673
RMS(Z_0)	1.087	2.706	1.644	1.587	1.215	1.510	1.509	1.497	1.428	1.051

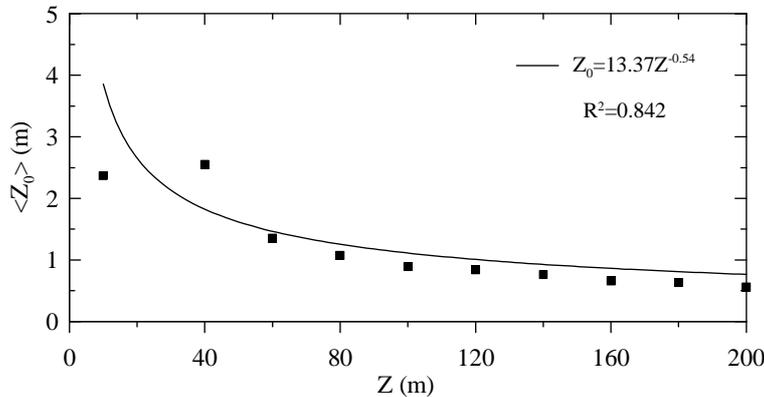


Figure 1. Vertical profile of $\langle Z_0 \rangle$ based on equation (2) (squares). The empirical law reported in equation (3) is also shown (line)

THEORETICAL EVALUATION OF THE WIND SPEED VERTICAL PROFILE

First of all, one can see that, rearranging equation (1) with $d_0=0$, it is possible to obtain a dimensionless form of the average velocity $AV(Z)$, viz.:

$$AV(Z) = U(Z) \frac{k}{U^*} = \ln\left(\frac{Z}{Z_0}\right) = \ln(Z) - \ln(Z_0) \tag{4}$$

Therefore, the coefficient of the first logarithmic term in the last hand side of (4) is equal to 1, while the bias is strictly linked to a Z_0 constant with height. In contrast, it is possible to show that a coefficient different by 1 is connected to a roughness length varying with height and linked to relationship (3).

In order to find a new formulation able to incorporate the above concept, the vertical wind profiles available from the field campaign in the case of neutral conditions have been carefully analyzed. The corresponding number of vertical profiles

relevant to that condition is equal to 1378. It has been considered a subset of 336 vertical profiles that match the following three conditions: (i) the wind direction is constant with height within the range of variation of 22.5 degrees, (ii) continuity of the measured data along the vertical profile in the height range 10+200 m and (iii) the functional relationship between wind speed and height is of logarithmic form. Careful inspection of the data set suggests that the wind profiles may be described by the following functional relationship:

$$AV(Z) = \alpha \cdot \ln(Z) + \beta \tag{5}$$

where $\alpha \neq 1$ and $\beta \neq 0$ are constant parameters to be determined. Equation (5) is a generalization of equation (4), and a relation between α and α_{Z_0} as well as between β and γ_{Z_0} exists. If Z_0 is independent of the height, by comparing equations (3), (4) and (5) it follows that α_{Z_0} coincides with the classical concept of roughness length and $\gamma_{Z_0} = 0$. In addition, it can be found that $Z_0 = \exp(-\beta / \alpha)$. For the sake of brevity, details on the previous sentences are not given here.

Table 2 shows the mean and the rms of α , β , the correlation coefficient R^2 associated to the individual velocity profiles and Z_0 . The corresponding vertical profile of the average dimensionless velocity is shown in Figure 2. The large value of the average correlation (0.74 ± 0.19) indicates that the logarithmic profile is a reasonable approximation of the observed profile. The parameter α is significant and its distribution is nearly symmetrical (not shown).

Table 2. Mean and rms of the parameters related to equation (5)

	α	β	R^2	Z_0 (m)
Mean	1.65	-2.72	0.74	5.30
rms	0.71	2.09	0.19	2.96

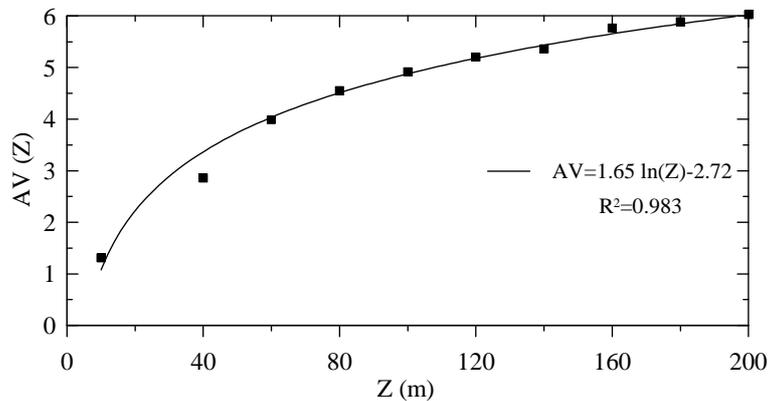


Figure 2. Vertical profile of the average wind speed.

By rearranging the previous equations it is possible to show that:

$$\begin{cases} \gamma_{Z_0} = -0.65 \\ \alpha_{Z_0} = 15.63 \end{cases}$$

Substituting the previous values in equation (3) yields:

$$Z_0 = 15.63 \cdot Z^{-0.65} \tag{6}$$

Relation (6) has to be considered as representative of the city of Rome, valid in neutral conditions for the height range between 10 and 200 m. Therefore, the vertical profile of the wind speed in neutral conditions is:

$$AV(Z) = 1.65 \cdot \ln\left(\frac{Z}{5.30}\right) \tag{7}$$

In dimensional terms:

$$U(Z) = \frac{1.65 \cdot U^*}{k} \ln\left(\frac{Z}{5.30}\right) \tag{8}$$

COMPARISON WITH THE CLASSICAL FORMULATION

The performance of the novel theoretical formulation (equation 7) is compared with that derived by the classical similarity theory (equation 1). In the latter, neglecting the displacement height, three different cases, each of one associated with a value of the roughness length, were considered, viz., $Z_0=0.33$, 1.0 and 3.0 m. Based on the average characteristics of the city of Rome, the first two are realistic values of Z_0 . In contrast, $Z_0=3.00$ m is not reasonable, but it is considered in the analysis in order to recognize the asymptotic behavior of the classical similarity theory. The comparison will focus on the average dimensionless wind speed $AVM(Z)$ as a function of the height:

$$AVM(Z) = \frac{\sum_{i=1}^{N_p} AV_i(Z)}{N_p} \tag{9}$$

where $N_p=336$. Hereinafter, the three cases of the classical similarity theory are indicated as AVM followed by the value of Z_0 , while the new formulation is indicated as AVM_NF (Figure 3).

Table 3. Comparison of the averaged dimensionless velocity as a function of the height obtained with the two theoretical laws

Z (m)	10	40	60	80	100	120	140	160	180	200
Measurements	1.44	3.15	4.35	5.08	5.58	6.07	6.41	6.85	7.10	7.23
AVM ($Z_0=0.33$ m)	3.78	5.32	5.77	6.09	6.34	6.54	6.71	6.86	6.99	7.10
AVM ($Z_0=1.0$ m)	2.55	4.09	4.54	4.86	5.11	5.31	5.48	5.63	5.76	5.87
AVM ($Z_0=3.0$ m)	1.38	2.92	3.37	3.69	3.93	4.14	4.31	4.45	4.59	4.70
AVM_NF	1.16	3.69	4.43	4.96	5.37	5.70	5.98	6.22	6.44	6.63

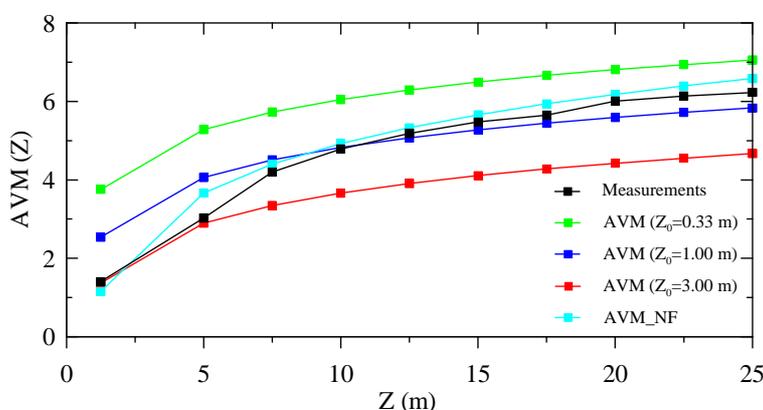


Figure 3. Vertical profiles of the averaged dimensionless velocity profiles calculated with the two theoretical laws

From the analysis of the dimensionless velocity calculated for different range of heights (10÷200 m and 10÷100 m) it follows that in the range 10÷200 m the classical formulation with $Z_0=0.33$ m shows velocity values significantly higher than those obtained for the other cases (Table 3). The comparison of the average differences obtained with the two theoretical laws is reported in Table 4. The model with minor differences within the entire vertical range is that based on the new formulation.

It is interesting to note that the classical similarity theory performs reasonably well at the higher levels for $Z_0=0.33$ m, while the lower altitude are modeled better by $Z_0=3.0$ m. This large value of the roughness length can be interpreted as the sum of the canonical roughness length plus the displacement height d_0 , not considered in the analysis. However, by considering other values of Z_0 the results do not improve significantly. This fact may be interpreted as a further evidence of the failure of the classical similarity theory based on a constant value of the roughness length. It is also interesting that the errors associated with the new formulation are nearly constant with height, in contrast to the classical law where the errors vary significantly along the vertical independently of the values of Z_0 .

Table 4. Comparison of the average differences obtained with the two theoretical laws. In red are highlighted the minimum differences for the two height ranges

Z (m)	10	40	60	80	100	120	140	160	180	200	10-200 m	10-100 m
AVM ($Z_0=0.33$ m)	5.48	4.69	2.01	1.02	0.57	0.22	0.09	0.00	0.01	0.02	1.19	1.66
AVM ($Z_0=1.0$ m)	1.24	0.88	0.04	0.05	0.23	0.58	0.87	1.50	1.79	1.85	0.95	0.70
AVM ($Z_0=3.0$ m)	0.01	0.06	0.97	1.94	2.72	3.74	4.44	5.75	6.31	6.41	1.80	1.07
AVM_NF	0.08	0.29	0.01	0.01	0.04	0.14	0.19	0.39	0.43	0.36	0.44	0.30

A second parameter for evaluating the accuracy of the models is the coefficient of determination R^2 between observed and theoretical profiles. The values reported in Table 5 confirm that the performance of the classical law for all the three cases is lower than that obtained by the new formulation.

Table 5. Coefficient of determination between the theoretical and the experimental wind velocity for the whole vertical profile.

R ²	10-200 m
AVM (Z ₀ =0.33 m)	-0.069
AVM (Z ₀ =1.0 m)	0.612
AVM (Z ₀ =3.08 m)	0.883
AVM_NF	0.896

SUMMARY AND CONCLUSIONS

The aim of this study was to investigate the applicability of the classical similarity theory in the case of urban environment. This implied the use of a meteorological data set acquired during the years 2005-2006 in a site located within the city of Rome, Italy. Vertical profiles of wind velocity and temperature acquired by a SODAR/RASS system up to 200 m agl as well as measurements taken by a routine meteorological monitoring were considered for the analysis. The investigation concerned statically neutral conditions. As expected, the results show a clear failure of the classical similarity theory. The use of the displacement height did not improve significantly the theory performance. An alternative formulation was therefore proposed which is based on the assumption of roughness length varying with height. The analysis shows a notable improvement of the results. In particular, the degree of agreements between observations and model is reasonably good for the whole thickness of the investigated boundary layer.

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