A Lagrangian stochastic model for estimating the high order statistics of a fluctuating plume in the neutral boundary layer

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Target

- Impact assessment of risks related to the dispersion of flammable gases and toxic substances.
- Simulation of the combined effects of the turbulent mixing and molecular diffusivity.
- Estimate of the concentration fluctuations and prediction of the higher statistics and the concentration PDFs.

References

- M. Cassiani, P. Franzese, U. Giostra, 2005a. A PDF micromixing model of dispersion for atmospheric flow. Part I: development of model, application to homogeneous turbulence and to a neutral boundary layer. Atmos. Environ. **39**, 1457-1469.
- J.V. Postma, J.D. Wilson, E. Yee, 2011a. Comparing two implementations of a micromixing model. Part I: wall shear-layer flow. Bound.-Layer Meteor. **140**, 207-224.
- J.E. Fackrell, A. Robins, 1982. Concentration fluctuations and fluxes in plumes from point sources in a turbulent boundary later. J. Fluid Mech. **117**, 1-26.
- B.L. Sawford, 2004. Micro-mixing modeling of scalar fluctuations for plumes in homogeneous turbulence. Flow Turbul. Combust. **72**, 133-160.

Current study

New experiment data set in wind tunnel:

- Measures of high order concentration statistics in a fluctuating plume in a neutral boundary layer.
- Poster session T8.

Numerical simulations:

- Comparison between experiments and computed solutions.
- Evaluation of the accuracy of the model.

Lagrangian Stochastic model

Equations describing the evolution of the position X_i and velocity U_i of a set of independent fluid particles.

$$dX_i = (\langle u_i \rangle + U_i')\, dt$$

$$dU_i' = \overbrace{a_i(\mathbf{X}, \mathbf{U}', t)\, dt}^{\text{deterministic term}} + \overbrace{b_{ij}(\mathbf{X}, \mathbf{U}', t)\, d\xi_j}^{\text{stochastic diffusive term}}$$

- U_i': Lagrangian velocity fluctuation related to the Eulerian mean velocity $\langle u_i \rangle$.
- a_i is estimated according to the well-mixed conditions[1].
- b_{ij} is defined from the Kolmogorov's hypotheses of self-similarity and local isotropy in the inertial subrange[2].
- $d\xi_j$ incremental Wiener process with zero mean and variance dt.

[1] D.J. Thomson, 1987. J. Fluid Mech. **210**, 529-556.
[2] S.B. Pope, 1987. Phys. Fluids **30**, 2374-2379.

Micromixing model

Molecular diffusivity is simulated by an Interaction by Exchange with the Conditional Mean (IECM) model.

$$\frac{dC}{dt} = -\frac{C - \langle C|u_i \rangle}{\tau_m}$$

- C is the concentration associated to a fluid particle and $\langle C|u_i \rangle$ is the mean scalar concentration conditioned on the local position and velocity.
- The micromixing time τ_m represents the temporal scale of the molecular diffusion:
  - parametrization of τ_m follows the formulation of Cassiani et al. (2005a)[3];
  - τ_m is assumed to be proportional to the time scale of the relative dispersion process, $\tau_m = \mu_t \tau_r$;
  - $\tau_m = f(\sigma_u, \varepsilon, \sigma_0, t)$

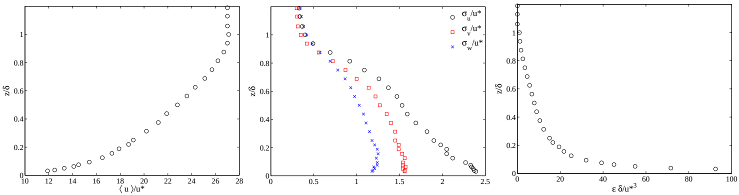
[3] M. Cassiani, P. Franzese, U. Giostra, 2005a. Atmos. Environ. **39**, 1457-1469.

SLAM: computational algorithm

1. **Pre-processing** (X and U):
   - simulation of the trajectories of an ensemble of particles released at the source location;
   - estimate of the conditional mean concentration $\langle C|u_i \rangle$ and the micromixing time τ_m.
2. **Simulation of the concentration fluctuations** (X, U, C):
   - instantaneous release of a uniform particle distribution in the domain;
   - initialization of the particle properties (X, U, C);
   - main time loop:
     - loop on all the particles:
       - update particle velocity and position;
       - apply boundary conditions;
       - update particle concentration;
     - update cell-centred statistics;
   - update time-averaged statistics.

Experimental set-up

Velocity field → Hot Wire Anemometry measures.



1) $\frac{\langle u \rangle}{u^*}$ vs $\frac{z}{\delta}$ 2) $\frac{\sigma_u}{u^*}$, $\frac{\sigma_v}{u^*}$, $\frac{\sigma_w}{u^*}$ vs $\frac{z}{\delta}$ 3) $\frac{\varepsilon \delta}{u^{*3}}$ vs $\frac{z}{\delta}$

- Boundary layer depth $\delta = 0.8$ m.
- Friction velocity: $u^* = 0.185$ m/s.
- Source height $\frac{h_s}{\delta} = 0.19$.
- Two source diameters: $\frac{\phi_1}{\delta} = 3.75\text{e-}3$, $\frac{\phi_2}{\delta} = 7.5\text{e-}3$.

Concentration field → measures of ethane (passive scalar) concentration by means of Flame Ionization Detector.

Numerical discretization parameters

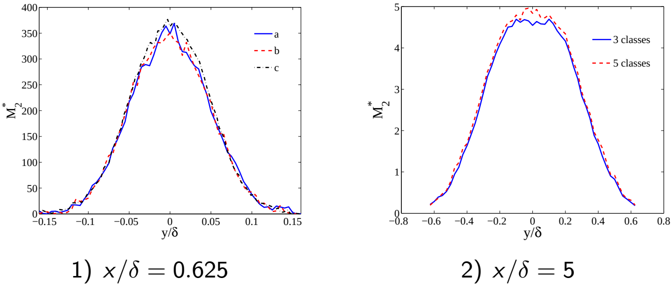


Figure: M_2^* vs y/δ

- **a**: $\Delta t = $ 1e-3, $\Delta x = 0.02$, $\Delta y = \Delta z = $ 5e-3;
- **b**: $\Delta t = $ 5e-4, $\Delta x = 0.02$, $\Delta y = \Delta z = $ 5e-3;
- **c**: $\Delta t = $ 1e-3, $\Delta x = 0.01$, $\Delta y = \Delta z = $ 3e-3.

Computational set-up

$\mathbf{C_0}$	σ_0	$\mathbf{C_r}$	μ_t	**velocity classes**
5.0	$\sqrt{2/3}d_s$	0.3	0.6	$3 \times 3 \times 3$

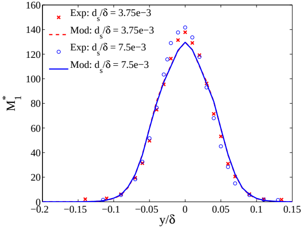
Table: Free parameter values adopted in the simulations

Non-dimensional concentration centred moments:

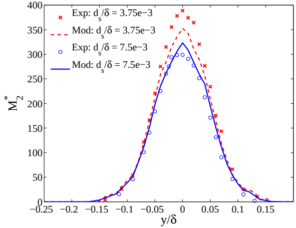
$$M_i^* = \left[\frac{1}{N_c}\sum_{p=1}^{N_c}(C_p - C_c)^i\right]^{1/i}\frac{u_\infty \delta^2}{Q} \qquad i = 1, 2, 3, 4$$

- u_∞: the velocity at the boundary layer height;
- N_c: number of particles in a discrete volume;
- C_c: mean concentration in a discrete volume;
- C_p: concentration associated to a particle.

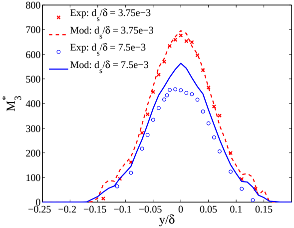
Results: M_i^* vs y/δ evaluated at the source height and $x/\delta = 0.625$



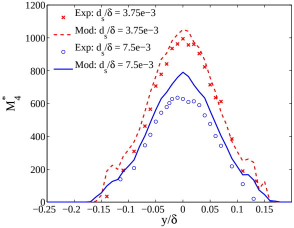
1) M_1^* vs $\frac{y}{\delta}$



2) M_2^* vs $\frac{y}{\delta}$

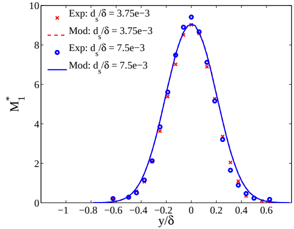


3) M_3^* vs $\frac{y}{\delta}$

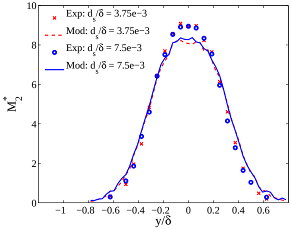


4) M_4^* vs $\frac{y}{\delta}$

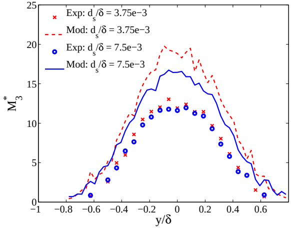
Results: M_i^* vs y/δ evaluated at the source height and $x/\delta = 3.75$



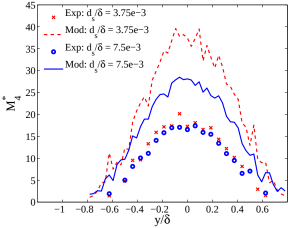
1) M_1^* vs $\frac{y}{\delta}$



2) M_2^* vs $\frac{y}{\delta}$

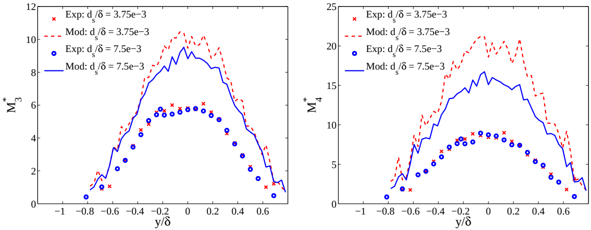


3) M_3^* vs $\frac{y}{\delta}$



4) M_4^* vs $\frac{y}{\delta}$

Results: M_i^* vs y/δ evaluated at the source height and $x/\delta = 5.0$



1) M_3^* vs $\frac{y}{\delta}$ 2) M_4^* vs $\frac{y}{\delta}$

$$D_{rel} = \sqrt{\frac{\int_{-\infty}^{\infty}\left[\left(M_i^*\right)_{S1} - \left(M_i^*\right)_{S2}\right]^2 dy}{\int_{-\infty}^{\infty}\left[\left(M_i^*\right)_{S2}\right]^2 dy}}$$

x/δ	D_{rel} M_3^*	D_{rel} M_4^*
3.75	0.17	0.36
5.0	0.12	0.29

Table: Relative difference of the third and fourth moments.

Conclusions

- The ability of the Lagrangian Stochastic Micromixing model SLAM to estimate concentration fluctuations was investigated.
- The dispersion of a fluctuating plume produced by a continuous release from a point source in a neutral boundary layer was simulated and a comparison with a new experimental data set was performed.
- Good agreement of the first four moments of the concentration close to the source.
- Good agreement of the mean concentration and variance in the far-field.
- Some discrepancies in the third and fourth moments of the concentration in the far-field.

Thank you for your attention!
Any questions?