ADAPTATION OF THE REYNOLDS STRESS TURBULENCE MODEL FOR ATMOSPHERIC SIMULATIONS

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15th Harmonisation within Atmospheric Dispersion Modelling for Regulatory Purposes Conference

Outline

1. Introduction and motivations
2. Reynolds Stress model
3. Atmospheric RSM constants
4. Validation of the model
5. Conclusions and perspectives

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1 – Introduction and motivations

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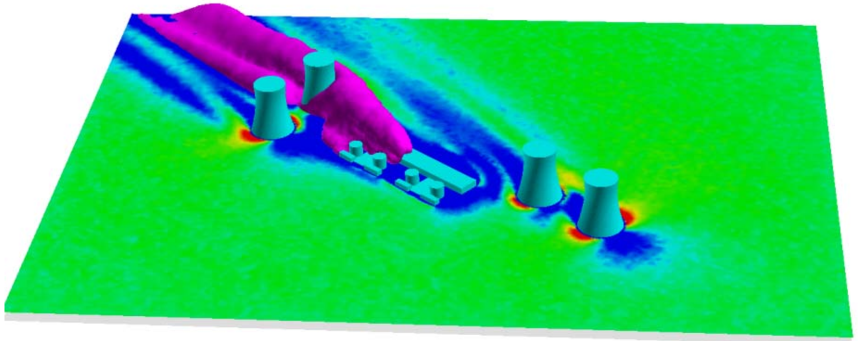
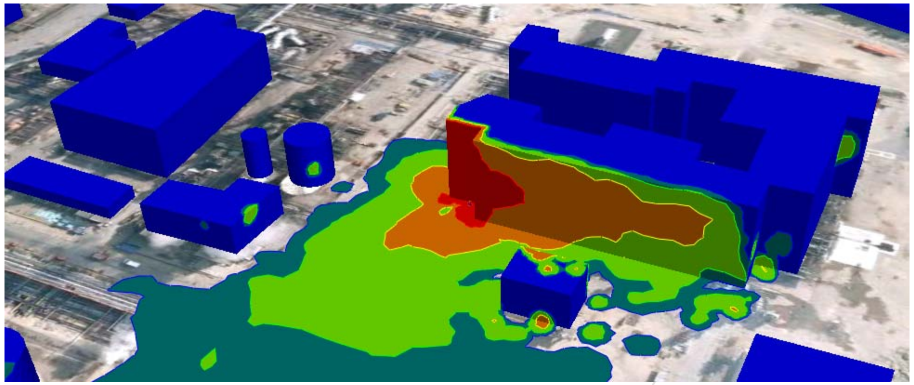
Research issues in atmospheric CFD modeling

- **Atmospheric CFD simulations (particularly RANS k-ε) are often used**
  - For local scale wind engineering
  - For dispersion in complex urban area with obstacles
  - For risk and safety assessment in industrial areas





1 – Introduction and motivations

Research issues in atmospheric CFD modeling

- **Different issues have to be properly solved**
  - Boundary conditions over the surface layer?
  - Anisotropy of the turbulence?
  - “Standard” or “atmospheric/Duynkerke” constants?
  - Value of the turbulent Schmidt number?

➔ Large uncertainty and user-dependent variability when comparing with field measurements

- **Objectives of this work**
  - Introduce anisotropy of turbulence using Reynolds Stress Model
  - Develop a 1D model for the entire Atmospheric Boundary Layer

➔ Provide parameterizations and boundary conditions for 3D CFD calculations

2 – Reynolds Stress model

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RSM equations

- **Reynolds Stress Model equations**
  - Reynolds Stress equation

$$\frac{\partial \overline{u_i'u_j'}}{\partial t}+\bar{u}_k\frac{\partial \overline{u_i'u_j'}}{\partial x_k}=\frac{\partial}{\partial x_k}\left(\frac{K_m}{\sigma_k}\frac{\partial \overline{u_i'u_j'}}{\partial x_k}\right)+P_{ij}+\phi_{ij}-\frac{2}{3}\delta_{ij}\varepsilon$$

$$\text{with}\quad P_{ij}=\overline{u_i'u_k'}\frac{\partial \bar{u}_j}{\partial x_k}+\overline{u_j'u_k'}\frac{\partial \bar{u}_i}{\partial x_k}$$

  - Turbulent dissipation rate equation

$$\frac{\partial \varepsilon}{\partial t}+\bar{u}_i\frac{\partial \varepsilon}{\partial x_i}=\frac{\partial}{\partial x_k}\left(\frac{K_m}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_k}\right)+C_{\varepsilon 1}\frac{\varepsilon}{2k}P_{ii}-C_{\varepsilon 2}\frac{\varepsilon^2}{2k}$$

  - Turbulent viscosity

$$K_m=C_\mu\frac{k^2}{\varepsilon}$$

2 – Reynolds Stress model

RSM constants

- **The preceding equations depends on 5 constants**

$$C_\mu\,,\ \sigma_k\,,\ \sigma_\varepsilon\,,\ C_{\varepsilon 1}\,,\ C_{\varepsilon 2}$$

- **Pressure-strain term ϕ_{ij}**
  - We choose the model of Gibson and Launder (1978)
  - This model introduces 5 other constants :

$$C_1\,,\ C_2\,,\ C_1'\,,\ C_2'\,,\ C_L$$

- **The "standard" values of these constants are:**

C_μ	σ_k	σ_e	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	C_1	C_2	C'_1	C'_2	C_L
0.09	1.0	1.3	1.44	1.92	1.8	0.6	0.5	0.4	0.39

2 – Reynolds Stress model

RSM constants

- **“Standard” constants are not adapted for the atmosphere**
- **Consider the Surface Boundary Layer**
  - For example, the C_μ constant controls the level of turbulent kinetic energy k:

$$B = \frac{k}{u_*^2} = \frac{1}{\sqrt{C_\mu}}$$

  - Wind tunnel measurements give B = 3.33 (i.e. C_μ = 0.09)
  - Atmospheric measurements (Panofsky and Dutton, 1984) give B = 5.48 (i.e. C_μ = 0.033)
- **It is necessary to define “atmospheric” constants:**
  - Duynkerke (1988) proposed a set of constants for the k-ε model
  - In this work, we propose a new set of constants for the Reynolds Stress Model

3 – Atmospheric RSM constants

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Determination approach

- **In the Atmospheric Surface Layer**
  - We assume a 1D Atmospheric Surface Layer, horizontally uniform
  - We identify the 1D RSM equations with the following analytical profiles

$$\bar{u}(z) = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right)$$

$$\varepsilon(z) = \frac{u_*^3}{\kappa z}$$

$$\overline{u'^2} = \alpha_x^2 u_*^2$$

$$\overline{v'^2} = \alpha_y^2 u_*^2 \quad \text{with} \quad \begin{cases} \alpha_x = 2.46 \\ \alpha_y = 1.9 \\ \alpha_z = 1.17 \end{cases} \quad \text{(Panofsky and Dutton, 1984)}$$

$$\overline{w'^2} = \alpha_z^2 u_*^2$$

$$\overline{u'w'} = -u_*^2$$

3 – Atmospheric RSM constants

Determination approach

- **It provides a system of equations for the constants:**

$\overline{u'^2}$ equation $$-C_1\frac{\alpha_x^2}{B}+C_1'C_LB^{\frac{1}{2}}\kappa\alpha_z^2-\left[\frac{2}{3}C_2\left(2-C_2'\kappa B^{\frac{3}{2}}C_L\right)-2\right]\frac{\kappa z}{u_*}\frac{\partial\bar{u}}{\partial z}+\frac{2}{3}(C_1-1)=0$$

$\overline{v'^2}$ equation $$-C_1\frac{\alpha_y^2}{B}+C_1'C_LB^{\frac{1}{2}}\kappa\alpha_z^2-\frac{2}{3}C_2\left(-1-C_2'\kappa B^{\frac{3}{2}}C_L\right)\frac{\kappa z}{u_*}\frac{\partial\bar{u}}{\partial z}+\frac{2}{3}(C_1-1)=0$$

$\overline{w'^2}$ equation $$-C_1\frac{\alpha_z^2}{B}-2C_1'C_LB^{\frac{1}{2}}\kappa\alpha_z^2-\frac{2}{3}C_2\left(-1-2C_2'\kappa B^{\frac{3}{2}}C_L\right)\frac{\kappa z}{u_*}\frac{\partial\bar{u}}{\partial z}+\frac{2}{3}(C_1-1)=0$$

$\overline{u'w'}$ equation $$\frac{1}{B}\left(C_1+\frac{3}{2}C_1'\right)+\left[C_2\left(1-\frac{3}{2}C_2'\kappa B^{\frac{3}{2}}C_L\right)-1\right]\alpha_z^2\frac{\kappa z}{u_*}\frac{\partial\bar{u}}{\partial z}=0$$

- **Which provides, after resolution, a set of “atmospheric” constants:**

C_μ	σ_k	σ_e	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	C_1	C_2	C'_1	C'_2	C_L
0.033	1.0	2.38	1.46	1.83	1.8	0.6	0.94	0.03	0.19

4 – Validation of the model

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Methodology of the 1D numerical model

- **We have developped a numerical 1D model for the ABL**
  - Flow is horizontally homogenous:

$$\overline{w} = 0 \quad \text{and} \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

  - Pressure gradient and Coriolis force in geostrophic balance
  - 1D equation model of mean wind speed, including Coriolis effect:

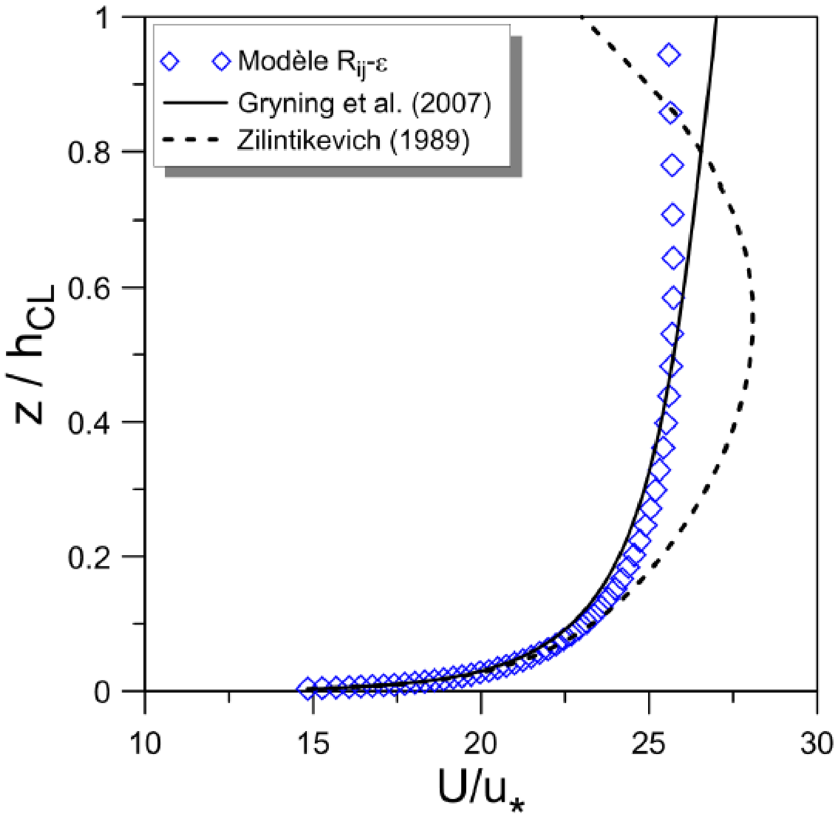
$$\frac{\partial \overline{u}}{\partial t} = -\frac{\partial \overline{u'w'}}{\partial z} + f\left(\overline{v} - V_g\right) \quad \text{and} \quad \frac{\partial \overline{v}}{\partial t} = -\frac{\partial \overline{v'w'}}{\partial z} - f\left(\overline{u} - U_g\right)$$

  - Reynolds Stress Model as turbulence closure model
  - Equations are solved numerically until a steady state
- **We apply the model on the overall ABL and compare it with empirical results**

4 – Validation of the model

Results in neutral conditions

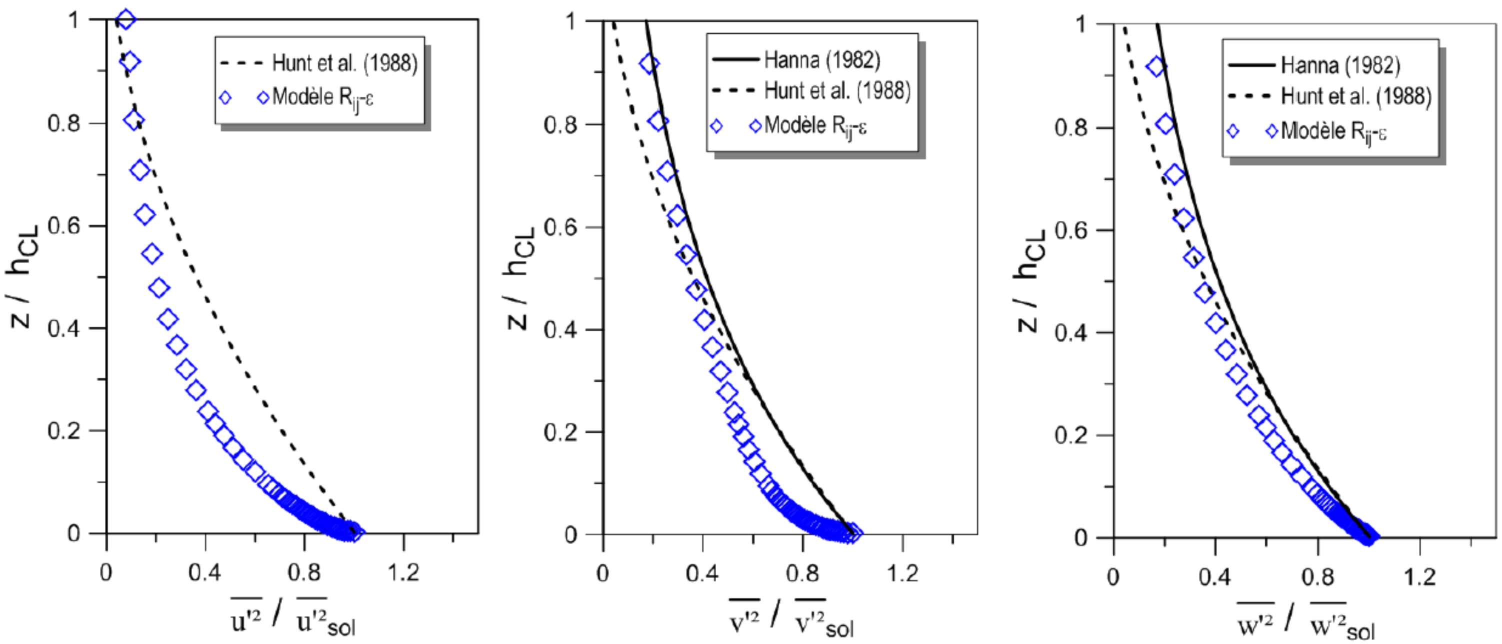
- **Velocity profile:**
  - For example for U_g =5m.s^{-1}, z_0 =0.01m, φ = 45°



4 – Validation of the model

Results in neutral conditions

- **Turbulence profiles (normalized by the ground value):**
  - For example for U_g =5m.s^{-1}, z_0 =0.01m, φ = 45°



4 – Validation of the model

Results in neutral conditions

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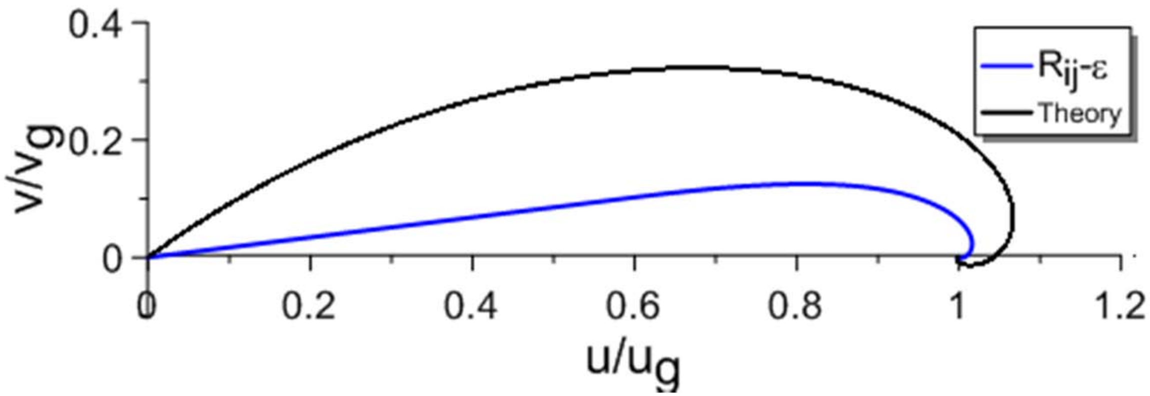
- **Ekman's theory predicts the twisting of the flow in the Ekman layer:**

$$\begin{cases} u = U_g\left[1 - \exp(-az)\cos(az)\right] \\ v = \mathrm{sgn}(f)\, U_g \exp(-az)\sin(az) \end{cases} \quad \text{with} \quad a = \sqrt{\frac{|f|}{2K_M}}$$

- **Simulation of the Ekman Layer with the 1D model:**



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4 – Validation of the model

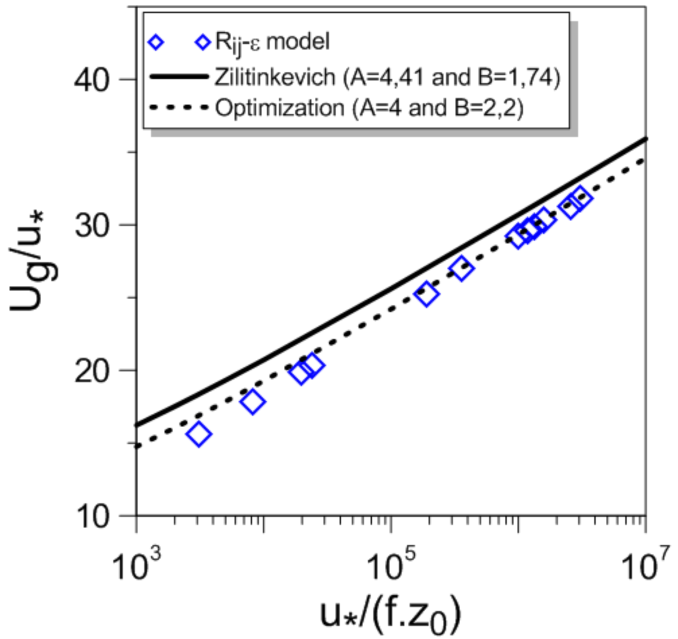
Results in neutral conditions

- **The Rossby similarity theory gives:**

$$\frac{U_g}{u_*} = \frac{1}{\kappa}\sqrt{\left(\ln\left(\frac{u_*}{fz_0}\right) - B\right)^2 - A^2}$$

A and B are empirical constants

- **Sensibility of the 1D model to control parameters U_g, f and z_0**

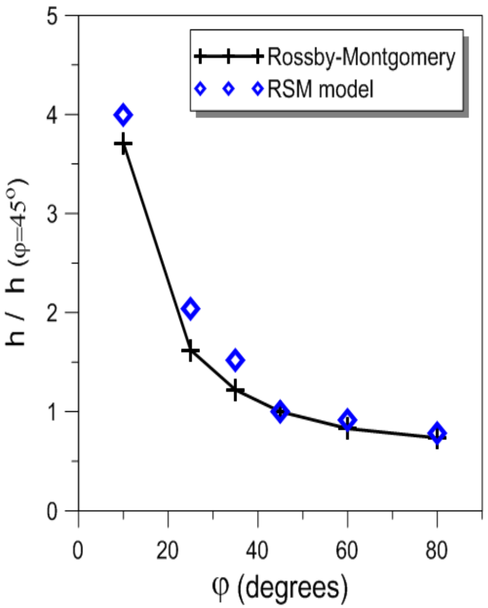


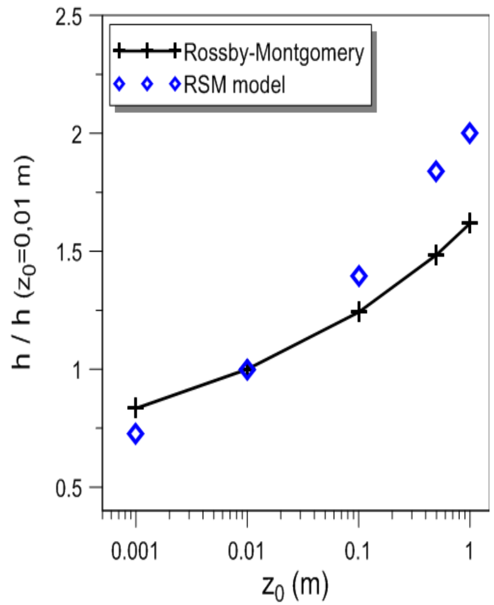
4 – Validation of the model

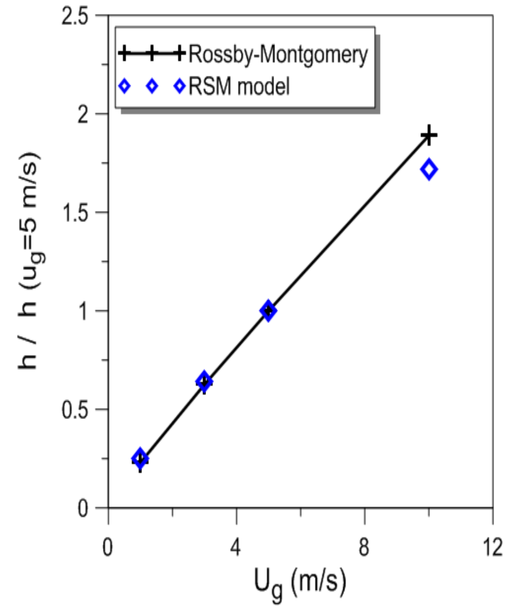
Results in neutral conditions

- **Atmospheric Boundary Layer height:**
  - The Rossby-Montgomerry equation

$$h = \frac{c u_*}{f} \quad \text{with} \quad c \approx 0.26$$



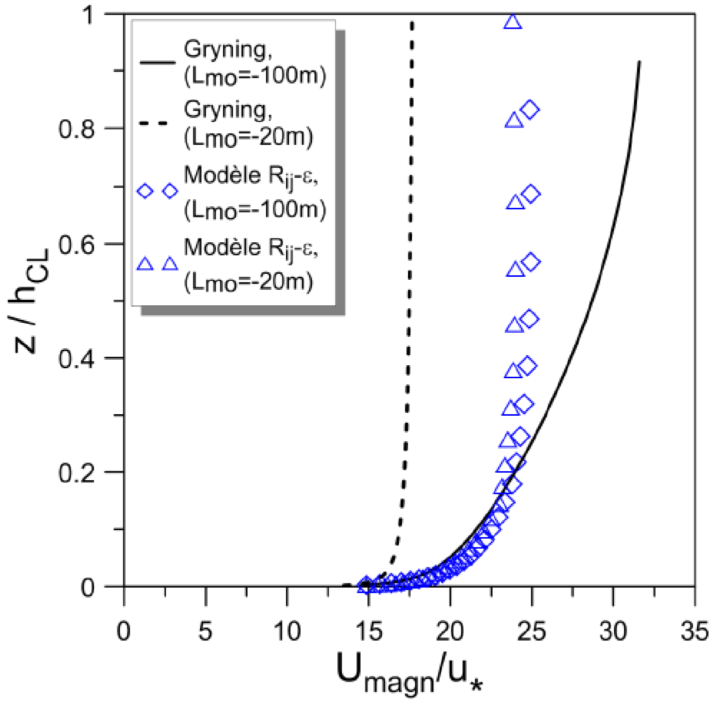




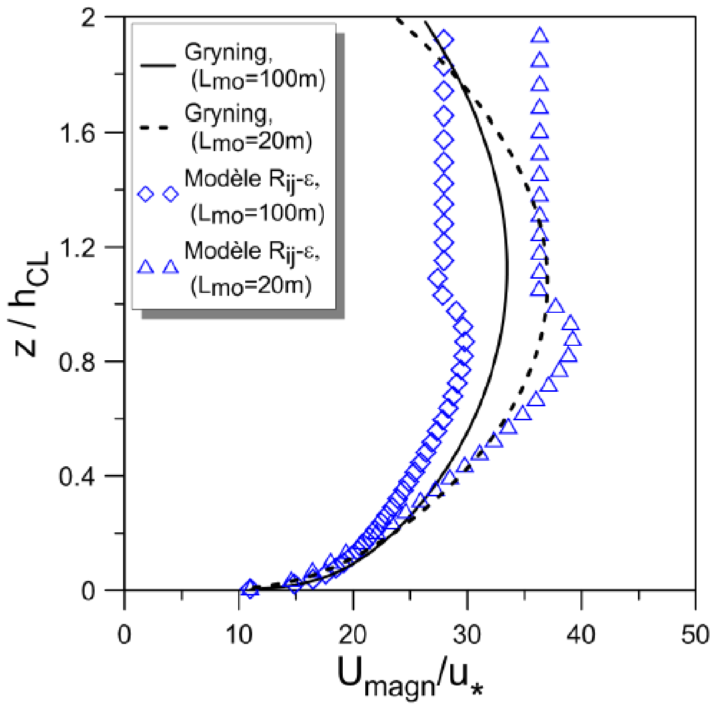
4 – Validation of the model

Results in stratified conditions

- **Velocity profile:**
  - For example for U_g =5m.s^{-1}, z_0 =0.01m, φ = 45°



Unstable conditions



Stable conditions

4 – Validation of the model

Results in stratified conditions

- **Turbulence profiles (normalized by the ground value):**
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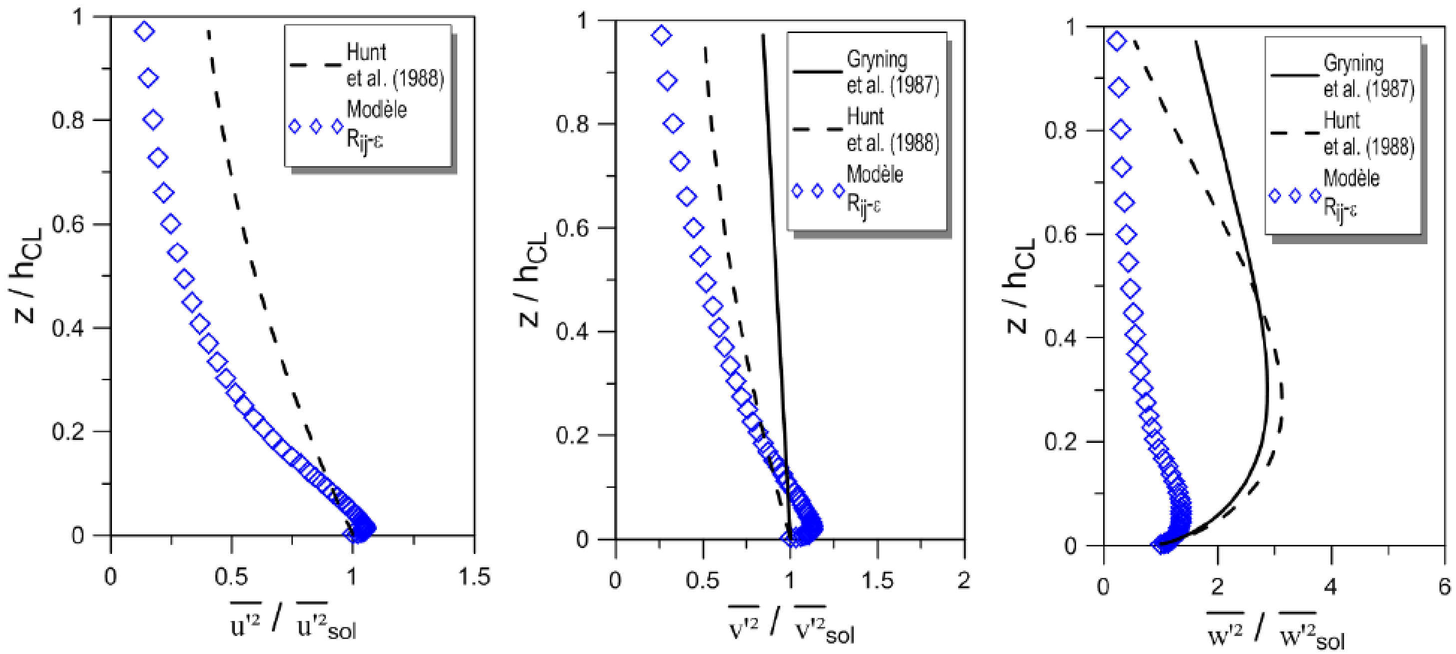

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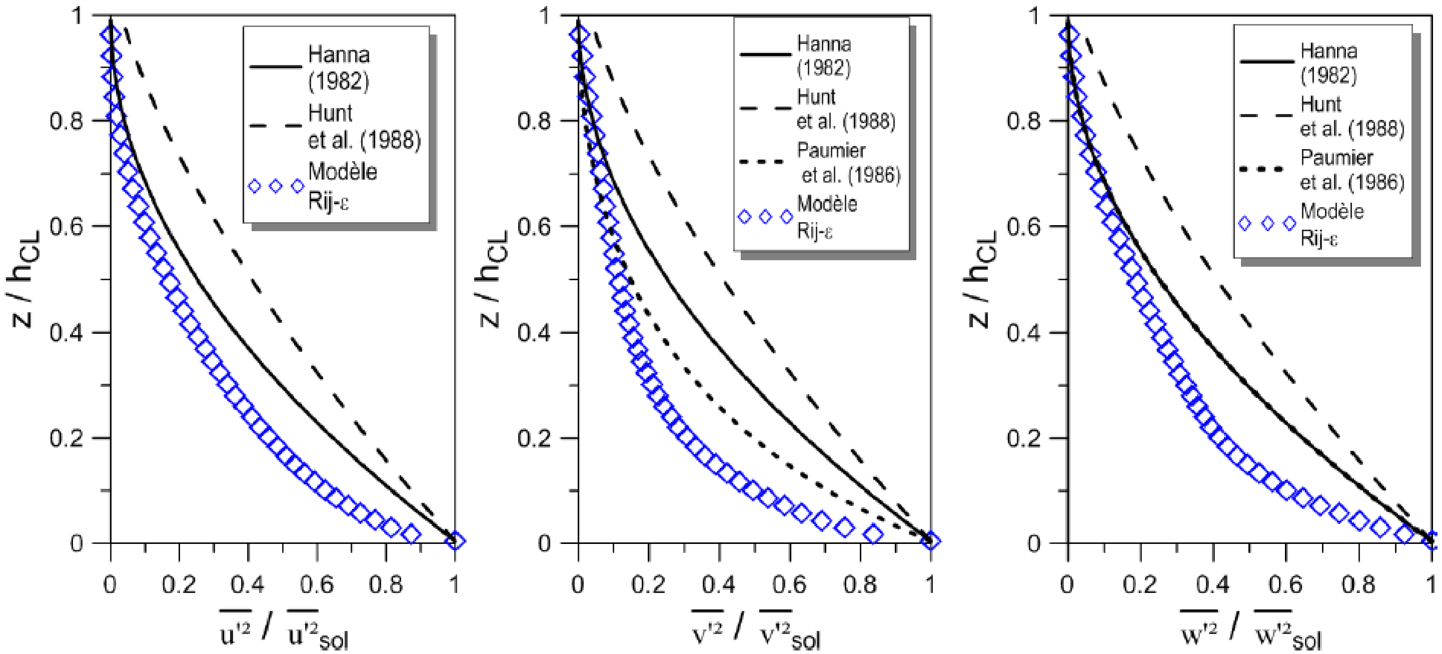

Stable conditions

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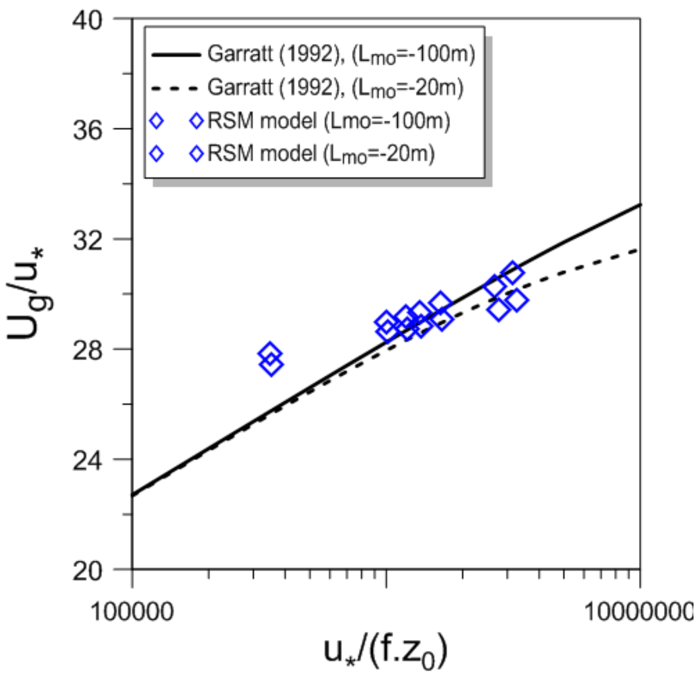
4 – Validation of the model

Results in stratified conditions

- **Relation between U_g and u_*:**

$$\frac{U_g}{u_*} = \frac{1}{\kappa}\sqrt{\left(\ln\left(\frac{u_*}{fz_0}\right) - B\right)^2 - A^2}$$

A and B are dependent on stability, measured empirically



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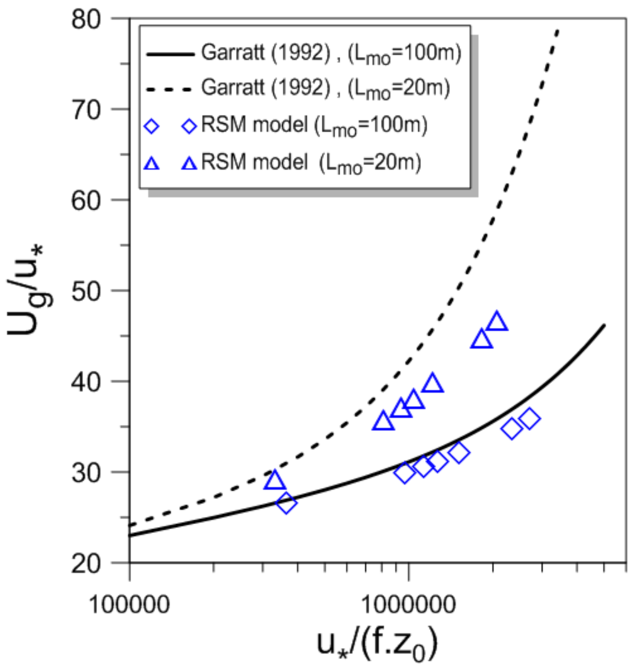

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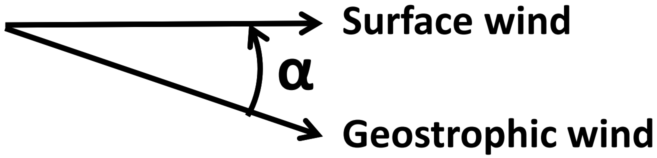
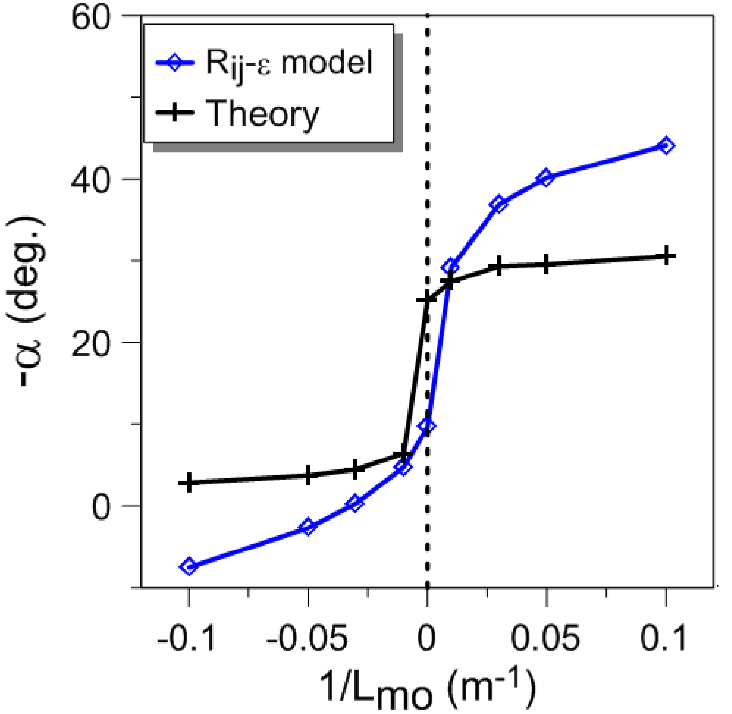


$$\sin(\alpha) = -\frac{A}{\kappa}\frac{u_*}{U_g}\,\mathrm{sign}(f)$$

A is dependent on stability, measured empirically



5 – Conclusions and perspectives

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- **Conclusions**
  - Development of a **new set of atmospheric constants** for the Reynolds Stress turbulence model
  - **1D simulations** of the Atmospheric Boundary Layer with this Reynolds Stress Model, **including Coriolis effects**
  - **Validation against empirical results**
- **Perspectives**
  - Validation in more complex configurations
  - Evaluation of the effect of anisotropy on dispersion modeling
  - Unified approach between “atmospheric” and “standard” RSM constants (see Poster H15-166)

Thank you for your attention ☺

Questions ?