RECONSTRUCTING THE HEIGHT OF AN ELEVATED POINT RELEASE IN LOW WIND STABLE CONDITIONS

Sarvesh Kumar Singh[1]*, Maithili Sharan*[2]*, Jean-Pierre Issartel*[3]*, Amit Kumar Singh*[2]*, Grégory Turbelin*[1]*,*
Amir Ali Feiz[1]*, Emerson Barbosa*[1]*, Pierre Ngae*[1]

[1]LMEE, Université d'Evry, 40 Rue du Pelvoux, 91020 Evry Cedex, France
[2]Centre for Atmospheric Sciences, IIT Delhi, Hauz Khas, New Delhi-110016, India
[3]Centre d'Etudes du Bouchet, 5, rue Lavoisier, BP 3, 91710 Vert le Petit Cedex, France

Abstract: In low-wind stable conditions, source reconstruction becomes sensitive to the height of the release and receptors measuring the concentrations. Therefore, an inversion technique, based on theory of renormalization, is proposed to identify the height of the release along with its location on the ground and emission rate in the atmosphere. The technique utilizes the information derived from the geometry of the monitoring networks. The retrieval algorithm is evaluated with real observations taken from Idaho diffusion experiment, conducted in low wind stable conditions.

Keywords: Point source reconstruction, Inverse modelling, Low-winds, Release height, Renormalization.

INTRODUCTION

Reconstruction of accidental point releases, or industrial disaster, at local scale is a challenging parametric estimation problem and especially concerned while evaluating the plume exposure over an urban region or implementing the emergency measures. Since the releases are hazardous and can not be measured on site, the estimation of their parameters (location on ground, height, emission rate) is addressed from distant measured atmospheric concentrations.

Often the accidents occurred in atmosphere, and their dispersion mechanism is subjected to their height above the ground level. The low-wind conditions are very frequent and the dispersion behavior of pollutants emitted from such elevated releases, vary due to frequent meandering, large wind variability and other features such as, lofting, fumigation, capping, fanning (Wark et al., 1998) etc. In such a case, the pollutants do not travel far from the source and exhibit a non-Gaussian plume distribution (Kumar and Sharan, 2009). This affects the resolution of the source reconstruction. In a recent study, Sharan et al. (2012) have shown that source parameters estimated during the inversion are sensitive to the height of the release and receptors in low wind stable conditions. In addition, approximating, or neglecting, height of the release and receptor raises an issue of model representativity with respect to the concentration observations, which further leads a discrepancy in the source estimates in the inversion. Thus, it is desired to estimate the height of an unknown elevated release along with its location and released mass for an effective decision-making and emergency strategy.

In particular, the reconstruction of point sources is relatively more difficult due to its representation on discrete model grids, high resolution, model representativity, measurement noise and singularity issues. The complexity increases further with increasing number of unknown source parameters.

In this paper, an inversion technique based on renormalization theory is proposed to identify the height of the point release along with its location on ground and emission rate. The applicability of the retrieval technique is shown with the real observations from Idaho diffusion experiment conducted at Idaho National Engineering Laboratory (INEL), USA (Sagendorf and Dickson, 1974) in low wind stable conditions.

RENORMALIZATION INVERSION TECHNIQUE

The study is focused for a continuous elevated point source emission. A total of n measurements $\mu_i(i=1,2,\ldots,n)$ are sampled at an altitude of height $z=z_r$ above the ground from an unknown elevated point source. The measurements are assumed linear to the point emission. Using an inner-product, a mutual correspondence between emission function $s(x,y,z)$ and μ_i can be defined on the domain Σ, by introducing the adjoint functions $a_i(x,y,z)$ such that (Marchuk, 1995; Sharan et al. 2009)

$$(s,a_i)=\int_\Sigma s(x,y,z)a_i(x,y,z)dxdydz=\mu_i \qquad (1)$$

The function $s(x,y,z)$ represents a continuous flux of emissions in unit amount of tracer per unit volume and time. Notice that a direct estimation of $s(x,y,z)$ using fundamental scalar product is not appropriate, as the adjoint function becomes singular at the position of the samplers and then the estimate may take the value either infinity or zero (Issartel, 2005). Therefore, a modified weighted scalar product is chosen to define the source-receptor correspondence (Sharan et al., 2009).

$$(s, a_{\varphi i})_\varphi = \int_\Sigma s(x,y,z) a_{\varphi i}(x,y,z) \varphi(x,y,z) dxdydz = \mu_i \tag{2}$$

In equation (2), $\varphi(x,y,z)$ is the weight function and the weighted adjoint functions $a_{\varphi i}(x,y,z) = a_i(x,y,z)/\varphi(x,y,z)$ are associated with the weighted scalar product defined as $(.,.)_\varphi$. The $\varphi(x,y,z)$ is defined in such a way that

$$\varphi(x,y,z) \geq 0 \qquad \text{and} \qquad \int_\Sigma \varphi(x,y,z) dxdydz = n \tag{3}$$

and is computed by an iterative algorithm described by Issartel (2005) and Issartel et al. (2007). This defines the ability of the monitoring network to see its environment and detect a possible release.

The emission function can be decomposed as $s = s_{\|\varphi} + s_{\perp\varphi}$ with parts respectively parallel and orthogonal to all $a_{\varphi i}$. The $s_{\|\varphi}$ is taken to estimate the measurements as in the scalar product, the contribution from the $s_{\perp\varphi}$ vanishes. The $s_{\|\varphi}$ can be written as a linear combination of $a_{\varphi i}$:

$$s_{\|\varphi}(x,y,z) = \sum_{i=1}^{n} \lambda_i a_{\varphi i}(x,y,z) \tag{4}$$

Using the concepts of linear algebra in equation (4), the source estimate is derived as (Issartel et al., 2007; Sharan et al., 2009)

$$s_{\|\varphi}(x,y,z) = \boldsymbol{\mu}^{\mathrm{T}} \mathbf{H}_\varphi^{-1} \mathbf{a}_\varphi(x,y,z) \tag{5}$$

in which $\boldsymbol{\mu}$ is the column vector of measurements and $\mathbf{H}_\varphi = (a_{\varphi i}, a_{\varphi j})_\varphi$ is the Gram matrix of the weighted adjoint functions. The superscript “T” denotes the transposition.

A point emission at (x_0, y_0, z_0) of intensity q is represented as a function $q\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)$ where $\delta(.)$ is the Dirac delta function. The measurements corresponding to it are

$$\mu_i = q\, a_i(x_0, y_0, z_0) = q\varphi(x_0, y_0, z_0) a_{\varphi i}(x_0, y_0, z_0) \tag{6}$$

Following the equation (6), an estimate for $s_{\|\varphi}(x,y,z)$ is derived as

$$s_{\|\varphi}(x,y,z) = q\varphi(x_0,y_0,z_0)\, s_0(x,y,z) \text{ with } s_0(x,y,z) = \mathbf{a}_\varphi(x_0,y_0,z_0)^{\mathrm{T}} \mathbf{H}_\varphi^{-1} \mathbf{a}_\varphi(x,y,z) \tag{7}$$

Using the Cauchy-Schwartz inequality, Sharan et al. (2009) have shown that the maximum of this source estimate (equation 5) in the whole domain coincides with the location of the point source. Thus, the source location (x_0, y_0, z_0) is retrieved from a given set of measurements by maximizing the function $s_{\|\varphi}(x,y,z)$ defined by equation (5). Once the location is obtained, the source intensity q is derived as (Sharan et al., 2012),

$$q = \frac{s_{\|\varphi}(x_0,y_0,z_0)}{\varphi(x_0,y_0,z_0)}. \tag{8}$$

DIFFUSION DATA

To evaluate the retrieval technique, experimental observations are taken for a trial (run 4) from Idaho diffusion experiments with light wind stable conditions (Sagendorf and Dickson, 1974) over flat terrain. The experiment involves a continuous release of tracer SF_6 from a stack of height of 1.5 m and concentrations sampling network of height of 0.76 m above the ground. The test criterion was a stable lapse rate with wind speed less than 2 m/s. 60 samplers were placed on each circular arc of radii 100, 200 and 400m from the point of release with an angular spacing of 6 degree between them. Meteorological measurements were given at 2, 4, 8, 16, 32 and 61 m level on a 61 m tower located on the 200m arc. The release rate in run 4 was 32000 µg/s and the release point was located at the centre of the circular arcs. However, the effective stack height is reported as 3m (Sagendorf and Dickson 1974).

NUMERICAL COMPUTATIONS

The adjoint functions a_i in the inversion technique, are chosen as retro-plume, computed by using an analytical dispersion model (Sharan et al., 1995), in backward mode assuming unit release from receptors. The corresponding dispersion parameters in horizontal and vertical directions are computed from the expressions of Luhar (2011) and Briggs (1973) respectively. The adjoint functions are computed with respect to each receptor.

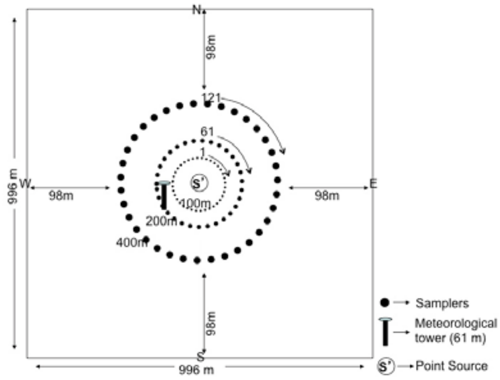

Figure 1: A 2-D layout of the computational domain (at the receptor's height above the ground) representing the receptor's arrangement. The size of the computational domain is 996m×996m. Black dots represent the receptors, located on the circular arcs of 100m, 200m and 400m. The number of receptors is counted from north in the clockwise direction. S' refers to the point source located at the centre of the circular arcs. Meteorological information's are provided by the meteorological tower (61m) located on the 200m arc.

For the inversion computations, a three-dimensional domain of size 996m$\times$996m×25m is chosen. In the horizontal direction, the domain is uniformly discretized into 499×499 grid points. Each mesh is taken as a square of 2m $\times$2m. In the vertical direction, the domain is discretized into 251 grid points with an uniform grid spacing of 0.1m. The centre of the circular arcs on the ground is at the grid point (250, 250). Since the effective stack height is reported as 3m and receptor's height is taken as 0.76m (Sagendorf and Dickson 1972), the ideal source location coincides with grid (250, 250, 31). Finally, convergence of the algorithm to compute φ with an error of 10^{-5} is attained with in 15 iterations.

RESULTS

The source estimation has been carried out for two types of data: (i) synthetic and (ii) real. The synthetic data are the concentration measurements generated from the same analytical model in the forward mode assuming experimental release parameters are known.

The corresponding weight function φ for the estimated height of release is represented in figure 2 panel (a). The φ is peaked at the position of the each receptor, and decays upwind of the monitoring network. A source located far downwind of the monitoring network cannot even be detected (Sharan et al., 2009) as the function φ vanishes immediately in that region. The isopleths of source estimates for both synthetic as well as real data at an estimated height of release, are exhibited in figure 2b and 2c respectively. The maximum of the source estimates are elongated along the mean wind direction. In panel (ii), only one lobe of maxima is observed with the synthetic data whereas two flat lobes are observed with the real data along the mean wind direction. The frequent occurrence of flat maxima lobes, are often associated with uncertainties. However in this case, the global maximum is observed at the centre of the monitoring network. The variation of source estimate with respect to the height above the ground level, for an estimated pair of grids (x, y) is shown in figure 3 for both synthetic and real data. In figure 3, the variation of source estimate is shown for a height of 5m above the ground level as the source estimate starts decreasing after that. It is shown that the source estimate becomes maximum at a height of 3m above the ground level with both synthetic and real case.

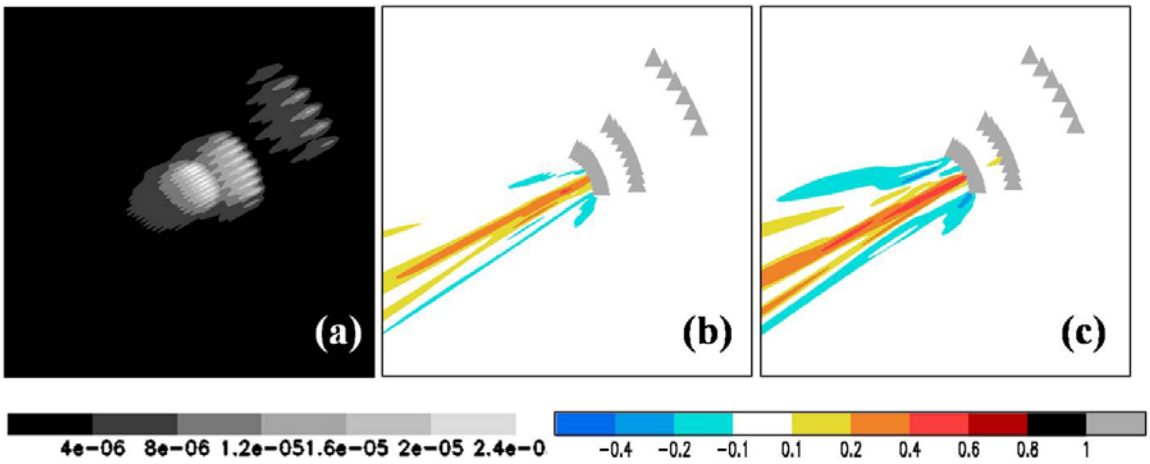

Figure 2: The isopleths of distribution of weight function (ϕ) (2a) and source estimates ($s_{\|\phi}$ in $\mu g m^{-2} s^{-1}$) (2b and 2c) at an estimated release height of 3m above the ground level. The panels (2b) and (2c) correspond to the source estimate contours with respect to the synthetic and real data respectively. The triangles denote the active receptors during the reconstruction.

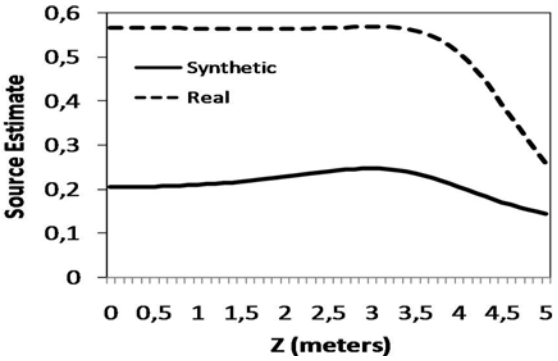

Figure 3: The variation of source estimate (in $\mu g m^{-2} s^{-1}$) with respect to the height above the ground level (z in meters) for a fixed set of grids (x, y). The dark smooth line corresponds to the synthetic data with fixed x=250, y=250 whereas a dotted line corresponds to the real data with fixed x=253, y=254.

With synthetic data, the height of release, location and emission rate are retrieved are almost similar to those prescribed (Table 1(a)). The global maximum of the source estimate coincides with the grid point (250, 250, 31) prescribed as the original location of the release. The height of the release is estimated as exactly 3m. The emission rate is also estimated as almost equal to the prescribed one, 32000 $\mu g / s$. The exact retrieval of the prescribed values with synthetic data proves the mathematical consistency of the proposed inversion technique.

With real data, the global maximum of the source coincides at the grid point (253, 254, 31) and obtained within an Euclidean distance of 10m (Table 1) from its experimental release location. The average height of the release is reconstructed as 3m, which is found as similar to the effective stack height during the experiment. In the reconstruction, the emission rate of the point source is slightly over-estimated (22%) within a factor of two. The estimation of location and intensity of release is affected with the errors caused by the noise and uncertainties associated with the measurements in diffusion experiment. In addition, the retrieval is influenced by the representativity errors associated with the dispersion model.

Table 1: Reconstruction results with the synthetic and real data. The experimental point releases (second row) and renormalized estimates with synthetic (third row) and real (fourth row) are indicated in terms of location in grid units (x, y, z), release height (hs in meters) and intensity (q) in $\mu g/\sec$. On the third column, the value of $s_{\|\varphi}$ in $\mu g m^{-2} s^{-1}$ at real source location is compared to the maximum value corresponding to estimated (Est.) source location. On the fourth column, the value of the weight function φ is compared at real and estimated (Est.) source locations.

	Release Parameters			$s_{\|\varphi}$		φ at ($\times 10^{-6}$)	
	(x, y, z)	hs	q	Real	Est.	Real	Est.
Experimental	(250, 250, 31)	3	32000	-	-	-	-
Reconstruction with Synthetic data	(250, 250, 31)	3	32000	1.02	1.02	31.9	31.9
Reconstruction with Real data	(253, 254, 31)	3	39156	1.36	1.42	31.9	36.3

CONCLUSION

The present study proposes an inversion technique, based on renormalization concepts, to identify an elevated release from a limited set of atmospheric concentration measurements. This addresses the typical problem of estimating the height of release along with its location on the ground and emission rate, in varying meteorological conditions. The technique utilizes the information available from the visibility of the monitoring network. The applicability of the inversion technique is shown in low wind stable conditions considering the real observations from Idaho diffusion experiment (Run 4). The inversion technique is evaluated with both synthetic as well as real data. With the synthetic data, the release parameters are identified exactly same as prescribed in original. With the real data, the source location is obtained within an error of 10m and the release height is estimated as 3m. The emission rate is also estimated with a slight over-estimation error of 22%, which corresponds to the model representativity errors and measurement uncertainties.

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