ON THE EFFECTS OF SAMPLING DURATION ON TURBULENCE MODELING AND ITS APPLICATION IN ATMOSPHERIC CFD SIMULATIONS

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Abstract: This paper investigates how to consider in numerical CFD modeling the influence of the sampling time of atmospheric turbulence characteristics. In wind-tunnel measurements, all of the turbulent time scales can be captured by a finite sampling duration because eddy sizes are limited by the spatial dimensions of the wind tunnel. In the atmosphere, the sampling duration is generally larger because the spectrum of velocity fluctuations is unlimited. This difference has led to the development of two sets of empirical constants for the RANS turbulence model: standard constants (generally applicable for small-scale simulations) and atmospheric constants. This paper aims a more general methodology for the simulation of turbulent levels.

We therefore present in this paper a model capable of calculating the turbulent characteristics for any given sampling duration, using the data from a simulation carried out by taking into account all time scales of atmospheric turbulence. We also demonstrate that this kind of simulation can be performed by using a RANS model, provided that a proper set of constants is taken into account. Such atmospheric constants are proposed by Duynkerke (1988) for the RANS k-ε model, and by Sadek et al. (2013) for the RANS Reynolds Stress Model. Finally, the new developed model is validated against atmospheric measurements on the SIRTA site, in France.

Key words: *Sampling duration, turbulence modelling, empirical constants, CFD simulations.*

INTRODUCTION

CFD codes have emerged as a powerful tool for simulating atmospheric flow in simple or complex terrain. Indeed, they generally offer a large choice of turbulence models, among which the RANS models are the most interesting because of the fact that they offer an interesting compromise between the precision of the result and the time of computation.

However, the use of RANS models, such as k-ε and RSM (Reynolds Stress Model), requires the empirical setting of values for constants figuring in their equations. These constants are of considerable importance because they set the overall level of turbulence to be simulated. The "classic" values for these constants have been set for wind tunnel experiments, which make them unsuitable for simulations of the atmospheric flow. Indeed, as was noted in Sadek et al. (2013), these constants, and mainly C_μ, are dependent on the level of turbulence B through:

$$B = \frac{k}{u_*^2} = \frac{1}{\sqrt{C_\mu}} \qquad (1)$$

where k is the turbulent kinetic energy (TKE) and u_* the friction velocity. The levels of turbulence B in wind-tunnel experiments are different from those in the atmosphere, as can be seen in table 1. Indeed, wind-tunnel measurements have given the classic value of B=3.33, whereas Panofsky and Dutton (1984) recommended B=5.5 through a large number of atmospheric measurements. This difference of levels of turbulence has given rise to two sets of empirical constants: "classical" and "atmospheric" constants. Based on Panofsky and Dutton (1984), Duynkerke (1988) proposed a set of constants for the k-ε turbulence model. In a similar way, Sadek *et al*. (2013) proposed atmospheric constants for the RSM model.

	B	C_μ
Wind-tunnel experiments	3.33	0.09
Atmospheric measurements	5.5	0.033

Table 1: Difference of turbulence levels and constants

However, there seems to be a lack of more general and thorough studies concerning the setting and the use of these empirical constants. For this purpose, this paper addresses the problem of developing a more global approach, which will allow unifying these two methodologies.

INFLUENCE OF THE SAMPLING DURATION

Objective

In order to propose a unifying methodology, the primary aim is first to understand the difference of turbulence levels between the two cases. In order to do so, we first present the theoretical definition of the standard deviation σ of a measured signal, which is defined as:

$$\sigma^2 = \int_0^{+\infty} E(n)\, dn \tag{2}$$

where n is the frequency and E is the spectrum of the wind's kinetic energy. However, the true value of the standard deviation of a given signal is never measured entirely, mainly due to the choice of a *finite* sampling duration τ. Consequently, this will induce a smaller value for σ than in the case of an infinite sampling duration. Thus, the value of σ is dependent on the choice of τ.

In wind-tunnels, the standard deviation σ will converge rapidly to a certain value because of the fact that the maximum sizes of the eddies are limited by the spatial dimensions of the wind tunnel. A finite τ can be used to obtain the final value of σ. This is not the case in the atmosphere: the spectrum of velocity fluctuations is unlimited, and consequently a longer sampling duration τ is required in order to capture the turbulent characteristics of the flow. This difference of τ, and consequently of the turbulent levels B, has led to the development of the so-called "classic" and atmospheric constants through equation 1. Thus, the constants are in fact dependent on τ:

$$\frac{k}{u_*^2} = \frac{1}{\sqrt{C_\mu}} = function(\tau) \tag{2}$$

Consequently, a more general approach between these two sets of constants can be obtained by presenting a model capable of estimating the influence of the sampling duration on the value of σ. We will present such model in what follows.

Theory and modeling

According to Pasquill and Smith (1983), and based on the work of Kahn (1957), the averaged standard deviation (over an infinite time) of a measured signal with a finite sampling duration τ can be given by:

$$\langle \sigma_\tau^2 \rangle_\infty = \int_0^{+\infty} E(n) \left(1 - \frac{\sin^2(\pi n \tau)}{(\pi n \tau)^2}\right) dn \tag{3}$$

Consequently, equation 3 allows us to quantify the effect of the choice of τ on the turbulence characteristics of a given flow. However, in order to solve this equation, an analytical form of the energy spectrum of E has to be specified. According to Wilson (1995), the longitudinal component of the energy spectrum of wind flow can be written as:

$$E_u(n) = \sigma_\infty^2 \frac{4T_{E,u}}{1 + \left(2\pi n T_{E,u}\right)^2} \tag{4}$$

where T_E is the Eulerian integral time scale. The vertical and transverse components are given by (with the assumption that they are equal):

$$E_v(n) = \sigma_\infty^2 \frac{4T_{E,v}\left(1 + 3\left(2\pi n T_{E,v}\right)\right)}{1 + \left(2\pi n T_{E,v}\right)^2} \tag{5}$$

However, there is no analytical solution to equation 3 along with the forms of the spectrum given in 4-5. Shoji and Stunkatani (1973) and Venkatram (1979), provide approximate solutions to these equations. By using their solution, we finally get the forms for the longitudinal, transverse and vertical components, respectively σ_u, σ_v and σ_w:

$$\frac{\sigma_u^2(\tau)}{\sigma_{u,\infty}^2} = 1 \quad 2\frac{T_{E,u}}{\tau}\left[1 \quad \frac{T_{E,u}}{\tau}\left(1 \quad exp\left(\quad \frac{\tau}{T_{E,u}}\right)\right)\right] \tag{6}$$

$$\frac{\sigma_v^2(\tau)}{\sigma_{v,\infty}^2} = 1 - 2\frac{T_{E,v}}{\tau}\left[1 - exp\left(-\frac{\tau}{T_{E,v}}\right)\right] \tag{7}$$

$$\frac{\sigma_w^2(\tau)}{\sigma_{w,\infty}^2} = 1 - 2\frac{T_{E,w}}{\tau}\left[1 - exp\left(-\frac{\tau}{T_{E,w}}\right)\right] \tag{8}$$

Equations 6-9 allow us to deduce the fluctuating components of the velocity of an atmospheric wind flow, for any given sampling duration, provided that we supply values of T_E and σ_∞ (which will be discussed later). We plot on figure 1 the form of the longitudinal component (equation 6). The transverse and vertical components have approximately the same shape.

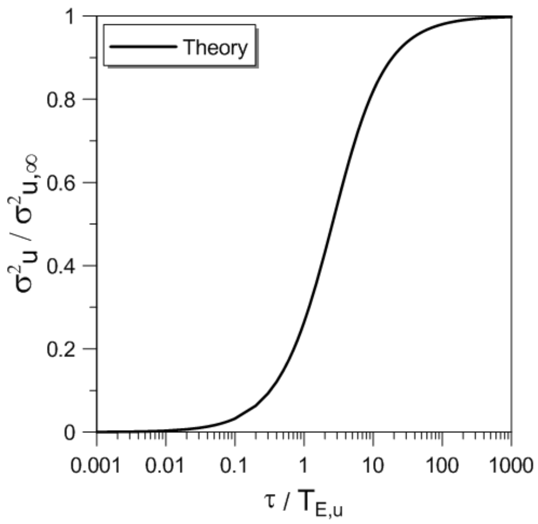

Figure 1: Dependence of σ_u on the sampling duration τ.

However, measuring the value of σ_∞ seems impossible because we would have to consider an infinite sampling duration. In fact, as it will be discussed later on, it is possible to choose a finite duration, but which is long enough in order to capture the majority of the time scales of atmospheric turbulence. This point will be explored in the next section. We also show that the value of σ_∞ can be calculated by a RANS k-ε or RSM models, provided that the empirical constants are selected in such way that the model can capture the overall levels of the observed turbulence. However, first, the aim is to validate the presented model (equations 6-8) against atmospheric measurements in order to justify its use in practical applications.

Validation of the model

In order to validate the model, we will use the database of measurements on the SIRTA site (Instrumental Site for Research on Atmospheric Remote Sensing – http://sirta.ipsl.fr), which is located near Paris. Measurements are made thanks to an ultra-sonic anemometer, with a frequency of 10Hz. The anemometer is located at an altitude of 10m above ground level. We must consider long enough sampling durations in order to cover different meteorological conditions. In addition, it is preferable to consider both summer periods (in order to cover unstable conditions) and winter periods (rather stable). Therefore, the measurement data spans three periods of one week:

- From 15 to 21 January 2010
- From 15 to 21 April 2010
- From 15 to 21 June 2010

Figure 2 shows the comparison between the atmospheric data and equations 6-8 concerning the three components $\sigma_i^2/\sigma_\infty^2$ of as a function of a nondimensionalized number τ/T_E. The value of σ_∞^2 is chosen at 1h (we will verify this hypothesis).

As we see on this figure, the measured values of σ^2 increase rapidly from $t/T_E \sim 0.1$, until reaching a higher plateau for $t/T_E \sim 100$ where they increase very slightly up to the value of σ^2_∞ at 1h. This stagnation indicates that we are in the presence of "the spectral gap", where wind flow fluctuates very little on time scales of about a one to a few hours. The stagnation was observed in the majority of the cases, and therefore we verify statistically the choice of σ^2_∞ at 1hour. In addition, this figure shows especially that the theoretical model approximates very well the atmospheric data, for a large number of meteorological conditions. Thus, assuming that we know, at a given point, the values of σ_∞ and T_E , equation 6-8 can be used to find the turbulent characteristics for any sampling duration.

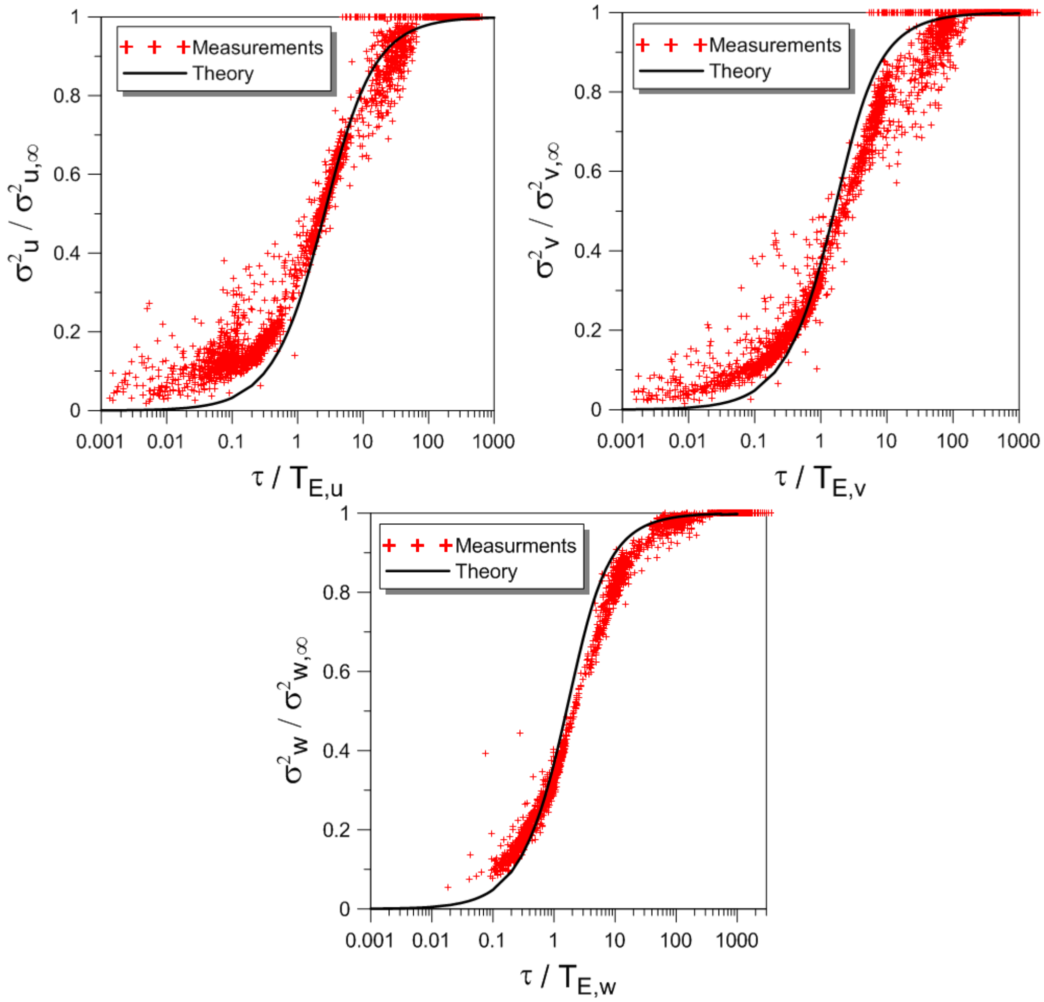

Figure 2: Dependence of σ_i on the sampling duration τ, according to theory and the measurements.

LINKING WITH THE k-ε AND RSM CONSTANTS

The values and of the levels of turbulence B at one hour (chosen as the infinite time), averaged on the entire span of 3 weeks of atmospheric measurements, are given in table 2.

	$\sigma_{U,\infty}/u_*$	$\sigma_{V,\infty}/u_*$	$\sigma_{W,\infty}/u_*$	$B = k_\infty/u_*^2$
SIRTA measurements	2.31	2.13	1.29	5.77
Panofsky and Dutton (1984)	2.39	1.92	1.25	5.48

Table 2: Comparison of turbulent levels

The measured values of ratios of anisotropy and of the turbulent levels, at the SIRTA site, are approximately the same as the values recommended by Panofsky and Dutton (1984), as can be seen on table 1. More precisely, the value of B of approximately 5.5 is therefore a good estimate of the levels of "small-scale" turbulence (one hour or less). On the other hand, the empirical constants for the RANS turbulence models, of Duynkerke (1988) for the k-ε model, and of Sadek et al. (2013) for the RSM model, were set using these levels of turbulence.

Therefore, one can calculate the values of σ_{∞} by using a RANS simulation, along with these atmospheric constants.

RECOMMENDATIONS

For a practical use of this methodology, we recommend simulating the atmospheric flow by using a k-ε or a RSM model, along with atmospheric constants. Then, the turbulent characteristics of the flow can be calculated for any given sampling duration less than one hour, by using equations 6-8. The local value for the Eulerian time scale can be given by:

$$T_E = T_L / \beta_i \quad (9)$$

where T_L is the Lagrangian time scale, and β_i is given by (Dosio et al., 2005):

$$\beta_i = C.U / \sigma_i \quad (10)$$

In equation 10, C is a constant, measured experimentally by Hanna (1981), who predicts a value of 0.7. Finally, the Lagrangian time scale can be calculated by using the data from a RANS simulation through equation 11 (Thompson, 1987), where C_0 is an empirical constant approximately equal to 2, and ε is the turbulent dissipation rate:

$$T_L = 2\sigma_i^2 / C_0 \varepsilon \quad (11)$$

CONCLUSION

In an effort to propose a more continuous approach between RANS simulations with either standard or atmospheric constants, we demonstrated in this paper that the main difference between these two methods lies in the fact that different sampling durations are considered in both cases, which leads to different levels of turbulence. In doing so, we presented, in this paper, a model capable of calculating the standard deviation of wind speed for any given sampling duration, thus achieving a continuous methodology.

However, the use of the developed model requires data from a simulation taking into account all time scales of atmospheric turbulence. We demonstrated that such simulation can be performed with a RANS k-ε or RSM turbulence models as long as the atmospheric empirical constants recommended by Duynkerke (1988) and by Sadek et (2013) are used (respectively for k-ε and RSM). In addition, the validity of this model was demonstrated through the comparison with atmospheric data spanning 3 weeks of meteorological conditions at a site at 10m above ground level. However, this model needs further validations. Indeed, in order to be applicable in RANS simulations, the model needs to be validated against data provided from vertical profiles covering the entire height of the atmospheric boundary layer, and not only in the surface boundary layer.

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