

ADAPTATION OF THE REYNOLDS STRESS TURBULENCE MODEL FOR ATMOSPHERIC SIMULATIONS

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Abstract: The RANS Reynolds Stress Model is one of the most comprehensive and powerful turbulence models available, allowing the simulation of the anisotropic nature of turbulence by solving transport equations for the Reynolds stresses. However, the generally used set of empirical constants, contained in the Reynolds stresses equations, are not appropriate for atmospheric simulations. We, therefore, present in this paper a new method for determining the empirical constants in order to achieve atmospheric levels of turbulence. A special focus is placed on the constants of the pressure-strain term, an important component contained in the Reynolds stress equation. We choose the Gibson and Launder (1979) linear modeling of the pressure-strain term, which is the one used in the commercial code Fluent. Results with the new set of constants are then validated against theoretical and empirical results of atmospheric conditions in flat terrain.

Key words: Reynolds stress turbulence model, empirical constants, CFD simulations.

INTRODUCTION

There is an increasing demand for CFD simulations of atmospheric wind flow for local scale applications, especially in the presence of complex terrain and obstacles. This is partly due to the fact that CFD codes generally offer a wide range of turbulence models. More precisely, the RANS (Reynolds Averaged Navier-Stokes) models have emerged as a powerful tool for simulating turbulence for practical and industrial applications. More complex models, such as LES (Large Eddy Simulations) are still used only for academic purposes, mainly due to the fact that they require large amounts of computing time.

In fact, the RANS k- ϵ is the most commonly used turbulence model, because it has proven to be an interesting compromise between quickness of the calculation and precision of the result. However, this model presents two main limitations. The first one concerns the hypothesis of isotropy that this model assumes, thus preventing the simulation of the highly anisotropic nature of turbulence, especially in the atmosphere. The second limitation concerns the empirical constants, which are very important in the simulation of proper levels of turbulence. The "standard" constants are generally used in wind-tunnel simulations, and do not allow reaching the generally higher levels of turbulence in the atmosphere. This second limitation has been tackled by Duynkerke (1998) by proposing atmospheric constants for the standard k- ϵ model.

A way of answering to the first limitation is by considering the use of the RANS Reynolds Stress Model (RSM), because it allows the simulation of the anisotropy of turbulence by solving transport equations for the Reynolds stresses. Therefore, we present in this paper a methodology of setting atmospheric values for the empirical constants figuring in the RSM model, in a way similar to the work performed by Duynkerke (1988) for the k- ϵ model.

METHODOLOGY

The RSM model solves six equations for the Reynolds stress terms ($\overline{u_i^j u_l^j}$), and one equation for the turbulent dissipation rate ϵ .

$$\frac{\partial \overline{u_i^j u_l^j}}{\partial t} + u_k \frac{\partial \overline{u_i^j u_l^j}}{\partial x_k} = \frac{\partial}{\partial x_k} \left(\frac{K_{\text{RSM}}}{\sigma_k} \frac{\partial \overline{u_i^j u_l^j}}{\partial x_k} \right) + P_{ij} + \phi_{ij} - \frac{2}{3} \delta_{ij} \epsilon \quad (1)$$

$$\frac{\partial \epsilon}{\partial t} + u_k \frac{\partial \epsilon}{\partial x_k} = \frac{\partial}{\partial x_k} \left(\frac{K_{\text{RSM}}}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial x_k} \right) + C_{1\epsilon} \frac{\epsilon}{2K} P_k - C_{2\epsilon} \epsilon \quad (2)$$

In these equations, we have:

- The turbulent viscosity: $K_{\text{RSM}} = C_\mu K^2 / \epsilon$.
- The production term: $P_{ij} = \overline{u_i^j u_l^j} \frac{\partial u_l}{\partial x_k} + \overline{u_i^j u_k^j} \frac{\partial u_l}{\partial x_k}$

For simplicity, we will neglect the terms concerning the thermal effects, Coriolis, and molecular diffusion (which, anyway, do not contain empirical constants). Except for the pressure-strain term ϕ , all the other terms on

the right side of equations (1) and (2) are similar to those of the standard k-ε model. Thus, we get the same 5 empirical constants as in this model: C_μ , σ_R , σ_ε , $C_{1\varepsilon}$ and $C_{2\varepsilon}$. On the other hand, the pressure-strain term ϕ is a very important component, because it sets the overall level of turbulence. Therefore, its modelling is also very important. Several authors, such as Lumley and Khajeh-Nouri (1974), and Launder *et al.* (1975), have succeeded in modeling this term. We choose the model proposed by Gibson and Launder (1978), as it seems the most complete to this date, mainly because it takes into account both the influence of the thermal effects and of the wall on the pressure field. This model figures in the CFD codes Fluent and OpenFOAM.

The use of the Gibson and Launder (1979) pressure-strain term requires the choice of values for 5 additional empirical constants (C_1 , C_2 , C'_1 , C'_2 and C_2). We therefore present in this paper a methodology of setting values for these constants in order to achieve atmospheric levels of turbulence, as no recommendation for these atmospheric constants can be found in the literature.

Setting the empirical constants

The methodology of setting values for these constant assumes a 1D flow (depending only on the vertical component z,) and directed along the x axis. In addition, it is also assumed that the flow is stationary, and in neutral conditions (thermal effects are neglected). In the surface boundary layer, the profiles of velocity and turbulence are thus written as:

$$\begin{aligned}
 & \bullet \quad u(z) = u_* / \kappa \ln(z/z_0) \\
 & \bullet \quad \sigma(z) = u_*^2 / (\kappa z) \\
 & \bullet \quad \overline{u'^2}(z) = \alpha_R^2 u_*^2 \\
 & \bullet \quad \overline{v'^2}(z) = \alpha_y^2 u_*^2 \\
 & \bullet \quad \overline{w'^2}(z) = \alpha_z^2 u_*^2 \\
 & \bullet \quad \overline{u'w'}(z) = -u_*^2
 \end{aligned} \tag{3}$$

where u_* is the friction velocity, z_0 the ground roughness length, and α_i constant terms for anisotropy. This gives for the turbulent kinetic energy (TKE):

$$k(z) = \frac{1}{2} (\overline{u'^2} + \overline{v'^2} + \overline{w'^2}) = B u_*^2 \tag{4}$$

B is the turbulence level, and is related to the constant C_μ trough:

$$B = \frac{1}{2} (\alpha_R^2 + \alpha_y^2 + \alpha_z^2) = \frac{1}{\sqrt{C_\mu}} \tag{5}$$

The usual value of $B = 3.33$ has been measured in wind-tunnel experiments, which consequently induces the “classical” value of $C_\mu = 0.09$. However, the turbulence levels measured in the atmosphere are substantially higher, with $B = 5.5$ according to the atmospheric measurements of Panofsky and Dutton (1984). Therefore, the value of C_μ should be set to **0.033**, as it is according to the work of Duynkerke (1988) for the k-ε model. The constants σ_R , $C_{1\varepsilon}$ and $C_{2\varepsilon}$ are kept equal to their “standard” values (see table 1), as it is approximately the case in Duynkerke (1988) for the k-ε model. In addition, the Rodi (1980) equation, which links the remaining σ_ε constant to C_μ , gives a value of $\sigma_\varepsilon = 2.01$ if $C_\mu=0.033$.

If we consider the assumptions given previously and equations (3), the only terms remaining in the equations of $\overline{u'_i u'_j}$ are the production, the pressure-strain, and the dissipation terms. Thus, we now have 6 equations for the remaining 5 unknown empirical constants of the pressure-strain term. In a similar way as in Gibson and Launder (1978), but by taking $B = 5.5$, we can set atmospheric values for these constants. These values are summarized in table 1.

C_μ	σ_R	σ_ε	$C_{1\varepsilon}$	$C_{2\varepsilon}$	C_1	C_2	C'_1	C'_2	C_2
0.033	1.0	2.01	1.44	1.92	1.8	0.6	0.94	0.03	0.19

Table 1: Summary of the atmospheric constants for the RSM model.

Implementation of the 1D numerical model

In order to simulate the entire atmospheric boundary layer (AB), a 1D model has been developed in order to provide vertical profiles for the 3D CFD codes in the case of a simulation of a local scale atmospheric flow. This

1D code solves the Navier-Stokes equations, along with a RSM turbulence model, by assuming horizontal homogeneity (the topography is flat and horizontally uniform). The ABL is simulated by taking into account the Coriolis force, and by assuming that the top of the domain is in geostrophic balance (the pressure gradients and the Coriolis force are in equilibrium). Thus, the solved equations are:

$$\frac{\partial u}{\partial t} = -\frac{\partial(\overline{u'v'})}{\partial z} + f(v - v_g) \quad (6)$$

$$\frac{\partial v}{\partial t} = -\frac{\partial(\overline{v'u'})}{\partial z} + f(u_g - u) \quad (7)$$

where f is the Coriolis force parameter and u_g and v_g are the geostrophic wind components. The Reynolds stress terms are solved with the RSM model (equation 1), and by assuming the aforementioned assumptions. The equations are resolved by iterating in time, until a steady state is reached (generally for about one hour). The vertical domain is 5km high (300m in the stable case), with an irregular mesh given by a geometric progression (70 vertical meshes, with refinement near the ground).

The comparison of this model with empirical and theoretical results, available in the literature, and in the presence of flat terrain, will allow us to study the performance of the RSM model, along with the new set of atmospheric constants, in the simulation of the ABL under various meteorological and ground type conditions. We will first focus on neutral conditions. Then, we will include the thermal effect term in the Reynolds stress equations in order to simulate stratified conditions.

RESULTS AND VALIDATION

Neutral conditions

The Rossby similarity equation in the Ekman layer links the geostrophic wind to the ground friction velocity, roughness length, and the Coriolis force:

$$\frac{U_g}{u_*} = \sqrt{\left(\ln\left(\frac{u_*}{f z_0}\right) - B\right)^2 + A^2} \quad (8)$$

where A and B are, in the neutral case, two constants set empirically by Zilitinkevitch (1989). Figure 1 shows that the RSM model is able to reproduce the trends predicted by this empirically fitted equation. The model points were obtained by taking different values of U_g , z_0 and f . These results also show a good sensitivity of the 1D RSM model to these input parameters.

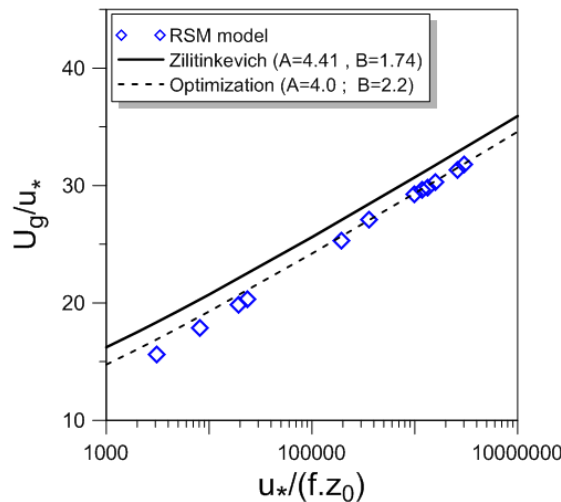


Figure 1 Comparison between the empirically fitted equation (8) and the RSM model results.

The Rossby-Montgomery equation allows estimating the ABL height h :

$$h = c u_* / f \quad (9)$$

with c being an empirical constant of approximately 0.26 (Garratt, 1992). In fact, there seems to be a high variability of this value because this equation is an extension of the stable case equation (10) to the neutral case.

Therefore, in order to compare in terms of trends rather than in exact values of h , we choose to plot, on figure 2, nondimensionalized values of h for several test cases. Thus, this figure shows a good sensitivity of h calculated by our 1D model h according to the domain latitude φ , the ground roughness length and the geostrophic wind, in a way very close to equation 9.

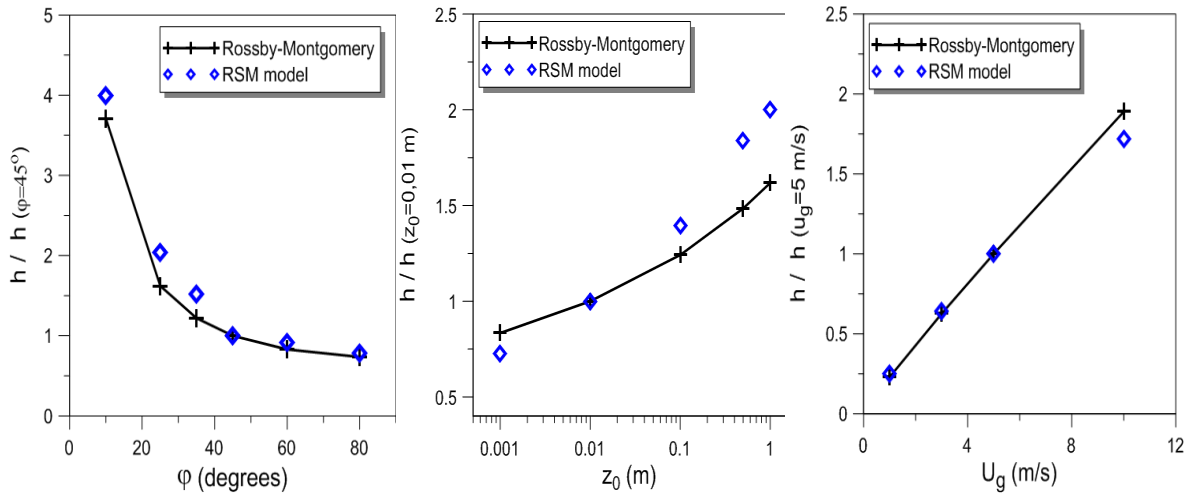


Figure 2 Comparison of ABL heights h as a function of different input parameters.

Stratified conditions

In equation 8, the A and B terms are no longer constants in stratified conditions but are dependent on the degree of stability of the atmosphere through the Monin-Obukhov length L_{mo} . Their forms have been set empirically and can be found in Garratt (1992). We display on figure 3 both unstable and stable conditions. We choose two types of stability: weak stratification (± 100 m) and rather strong stratification (± 20 m). As figure 3 shows, we find very good agreement between the two methodologies: the 1D RSM model is able to reproduce the ABL even in strong stratification.

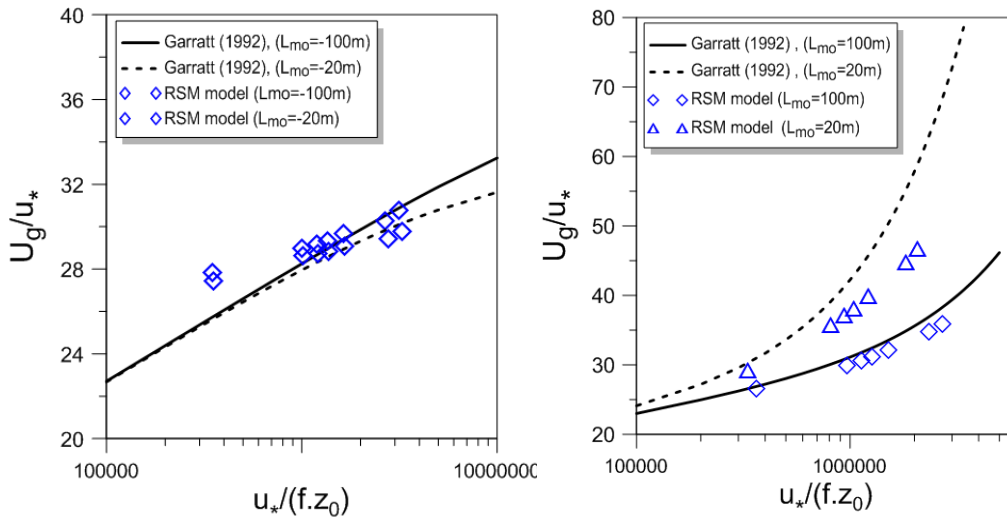


Figure 3 Comparison between the empirically fitted equation (8) and the RSM model results for unstable (left) and stable conditions (right).

The Rossby similarity also allows estimating the twisting of the flow in the Ekman Layer (i.e. the Ekman spiral). Equation 10 gives the angle α between the direction of the flow at the top of the ABL and at the ground-level, (the latter being defined as an east wind and α is in a counter clockwise direction):

$$\sin(\alpha) = -\frac{A u_*}{\kappa U_g} \operatorname{stgr}(f) \quad (10)$$

In this equation, A is the same empirical function as the one appearing in equation 8. Figure 4 shows similar trends between this equation and the 1D RSM model, for values of L_{mo} ranging from very unstable to very stable conditions. However, the difference between these two methodologies increases as the stratification increases. This difference is especially high in the stable case; however, the empirical function A is an average of measurement values of high variability, as can be seen in Nieuwstadt and van Hop (1981). Nevertheless, we have shown that the 1D RSM model has succeeded in predicting a good sensitivity of the flow twisting according to the stratification.

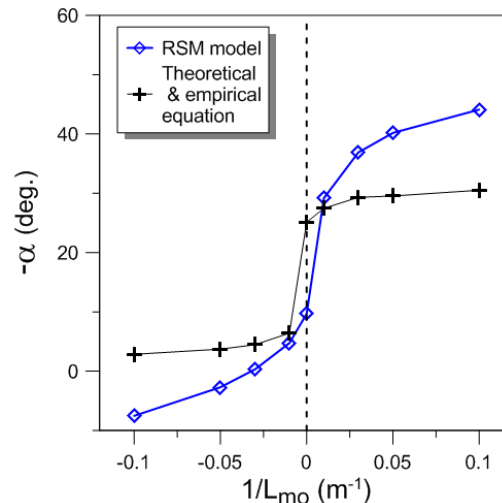


Figure 4 Angle difference between the direction of flow at the top of the ABL and at the ground.

CONCLUSION

First, we discussed in this paper the necessity of using the RSM model in order to simulate the highly anisotropic nature of turbulence for atmospheric simulations, especially in complex terrain. Therefore, we presented a methodology able to set values for the empirical constants in order to enable the RSM model to simulate atmospheric levels of turbulence. More precisely, we succeeded in setting values for the constants appearing in the important pressure-strain term, modelled by Gibson and Launder (1978).

We then validated the new set of atmospheric constants by using a 1D CFD code, along with a RSM model, and by comparing its results with several empirically fitted equations (available in literature) from measurements on flat ground. This comparison showed very good agreement between these two methodologies, for a large number of input values and for neutral, stable and unstable conditions. Therefore, we succeeded in demonstrating the validity of the new set of atmospheric constants for the RSM model. However, further validation of this methodology should include atmospheric wind flow measurements in the presence of a complex topography (e.g. a 3D hill), in both neutral and stratified conditions.

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