Using a power-exponential function to classify CO_2, PNC_{tot}, and student presence in a classroom in Rome

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1. Introduction

In the framework of the VIEPI project, an experimental campaign was conducted in a classroom of the Department of Environmental Biology, University of Rome 'La Sapienza', to investigate the link between pollutant concentration and student attendance. Details of indoor sites is found in Pelliccioni & Gherardi [1]. The campaign was conducted over two periods of approximately 30 days each, namely, October-November 2021 and March 2022. Here we show results for the March 2022 indoor field campaign. The measured variables included CO_2 concentration, air temperature (T), relative humidity (RH), particle number concentration (PNC_{tot}), average particle diameter (D_{av}), and student number (N_s). Classrooms were always naturally ventilated.

The purpose of this work is to demonstrate how a generalization of the Modified Gamma Distribution (MGD) [2], the power-exponential function (I_{powExp}), can be used to understand and explain the behavior of CO_2 in relation to student presence and to reproduce the dimensional distribution of ultrafine particulate matter. While MGD is based on a two-parameter approximation, I_{powExp} represents a fully parameterized three-parameter model.

The results show how the parameters of this distribution effectively describe the accumulation and depletion phases of indoor CO_2 due to student presence, and how the same function adequately reproduces the values of the dimensional distributions of indoor ultrafine particulate matter.

2. Material and methods

2.1 Definition of IpowExp function

The explanation of indoor pollutant can be improved using a function composed by the product between a power function and an exponential function:

$$V(t) = V_0 \cdot p^{K_{pow}} \cdot exp^{k_{exp} \cdot t} \tag{1}$$

In such cases, the dependent variables V concern the measurements of CO_2 inside the indoor classroom. The carbon dioxide has been used as a tracer of human emission of breathe in the classroom and indoor ventilation (Persily [3]). These measurements have been used to estimate the time response in the indoor connected with the presence of students. For such CO_2 measurements, the equation (1) transforms as:

$$CO_2(t) = CO_2^0 \cdot t^{K_{pow}^{CO2}} \cdot exp^{K_{exp}^{CO2} t} \tag{2}$$

and expresses the time evolution of the observed CO_2 correlated with the actual presence of students. The coefficient K_{exp}^{CO2} in eq (2) is expressed in t^{-1} unit, while the coefficients K_{pow}^{CO2} is no-dimensional number. The parameters of PowExp function K_{pow} and K_{exp} are always calculated by a best fit on own data. Whereas MGD models impose a constraint on the values of the parameters K_{pow} and K_{exp} - typically requiring the former to be positive and the latter negative - the I_{powExp} formulation allows the parameters to vary freely, including cases where both share the same sign. In such scenarios, the resulting signal may be interpreted as a dominant component (e.g., an exponential forcing term) augmented by a secondary power-law contribution. Empirically, in physical systems, the parameters often exhibit opposing signs, with K_{pow} generally positive and K_{exp} negative.

2.2 Statistical observation of main variables

The measured variables included CO_2 concentration, air temperature (T), relative humidity (RH), particle number concentration (PNC_{tot}), average particle diameter (D_{av}), and student number (SN). CO_2 has been monitored in a classroom at 1 minute resolution, averaging between 7/3/2022 up to 10/4/2022 for a total amount of about 44350 data collection. CO_2 Total database concerns about 33 days of one minute time resolution (two days are lacked because no CO_2 has been collected). The hourly average during the weekdays and weekends are shown in following Table 1.

week	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Mon	430	430	431	431	431	433	438	450	479	942	1381	1348	1202	827	1198	1457	1261	1002	743	568	509	480	467	460
Thr	457	454	450	449	449	450	452	459	475	582	706	653	631	581	705	973	978	935	701	561	512	480	465	459
Wed	455	454	455	455	453	454	456	461	483	871	1200	1270	1292	986	1196	1268	784	572	505	483	470	465	463
Thu	462	459	455	453	451	449	450	454	486	688	857	948	1083	853	806	839	711	643	555	506	484	474	467	464
Fri	462	460	459	457	454	454	456	461	473	474	487	643	845	753	961	1159	888	709	600	546	509	487	472	464
Sat	458	453	450	448	447	445	445	448	448	445	440	437	435	433	431	429	427	425	424	426	429	433	438	441
Sun	443	443	433	439	437	435	435	437	437	436	434	430	427	424	423	420	415	406	409	414	417	421	422	425

(a)

week	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Mon	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	69,2	54,2	39,3	41,4	0,0	60,7	57,9	16,2	19,6	9,2	0,0	0,0	0,0	0,0	0,0
Thr	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	13,2	13,6	10,1	9,2	0,2	32,3	34,2	24,6	23,2	0,2	0,0	0,0	0,0	0,0	0,0
Wed	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	50,8	40,6	44,9	45,2	1,7	60,6	49,0	8,8	7,6	0,1	0,0	0,0	0,0	0,0	0,0
Thu	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	26,8	22,5	28,9	27,3	0,0	25,1	18,4	16,0	14,1	0,0	0,0	0,0	0,0	0,0	0,0
Fri	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	26,9	24,9	0,0	46,0	37,1	6,6	3,3	0,0	0,0	0,0	0,0	0,0	0,0
Sat	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
Sun	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0

(b)

Table 1: Daily Mean CO2 (a) and Np (b) observed during weekdays

The table show that the maximum values of CO_2 verified during the working days in coincidence with the presence of students (from 9:00 to 18:00) with 885.6 ppm, while for the same hours during the weekend the value is about 427.5 ppm, totally coincident with the average values without students: the human contributions are evident by comparing the observed values with those measured during the daily time.

2.3 Classification Days in relationship with the presence' students

Given the massive amount of measured data, which is challenging to synthesize solely with a model, a classification method based on student presence has been used to facilitate and clarify the proposed modeling application. In order to show the results on the application of PowExp function on the explaining the indoor CO_2 pollutant, a classification of hourly data base on the actual presence of students has been needed. To identify representative days, a classification system was developed based on the following criteria:

-         Type A: Days without lessons (10 days)
-         Type B: Days with a few consecutive hours of lessons (4 days)
-         Type C: Days with a constant average presence of students (10 days)
-         Type D: Days with complex patterns of student presence (11 days)

The selection of events was based on a classification of each day considering the time shift 9:00 AM - 6:00 PM (coinciding with lesson time).

Type A refers to events during the weekend (Sunday and Saturday), when no lessons were conducted, and all doors and windows were closed. Type B concerns events where student presence was observed for a maximum of one to two hours. Type C pertains to events where the number of students remained constant throughout the lesson hours. Type D refers to events with complex patterns of student presence, characterized by variability in both the number of students and the time intervals. For each representative day, a modeling analysis was conducted.

3. Results and discussion

3.1 Observed values of the model variables during class hours

The measured values of CO_2, PNC_{tot}, temperature (T), and occupancy (N_s) during the selected periods used to classify the days are shown in Figure 1. As clearly observed, CO_2 concentrations for type A events are at their minimum detectable levels, namely 429 ppm, which is only slightly above the typical outdoor background level of approximately 420 ppm. CO_2 levels increase progressively, with average concentrations of 704 ppm, 835 ppm, and 1050 ppm for classes B, C, and D, respectively. These trends are fully consistent with the average number of students present in the classroom, which were 0.0, 9.8, 20.9, and 34.7 for classes A, B, C, and D, respectively.
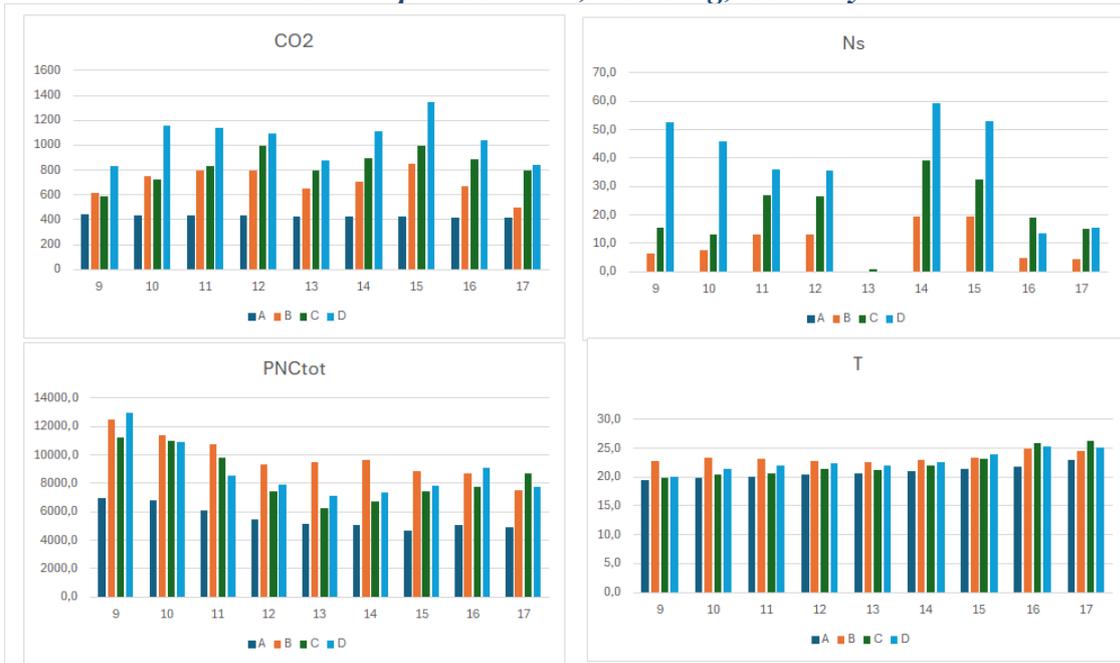
Figure 1: *Mean values of CO_2, total particle number concentration (PNC_{tot}), occupancy (Ns), and temperature (T) for classes A, B, C, and D*

Each student appears to contribute approximately 14.7 ppm of CO_2. In contrast, no clear proportional relationship is observed between ultrafine particle (UFP) concentrations and the number of students present. Indoor temperature reaches its maximum during type B events (T = 23.4 °C), while the minimum value of 20.3 °C is recorded during type A events, when classrooms are closed and there is no heat exchange with the outdoor environment.

3.2   Results obtained using the I_{powExp} function for CO_2

In addition to the parameters associated with the I_{powExp} function, each simulation also includes the total duration of the peak (T_d), expressed in minutes. This parameter represents the time required for CO_2 levels to return to background values, prior to the events induced by human presence. The following days were selected as representative of the analysed periods:

A	B	C	D
27/3/22	11/3/22	24/3/22	28/03/22 *(M)*
2/4/22	22/3/22	31/3/22	28/03/22 *(A)*
3/4/22	29/3/22	7/4/22	4/04/22 *(M)*
			4/04/22 *(A)*
			8/04/22 *(M)*
			8/04/22 *(P)*

For classes A, B, and C, the CO_2 time series data encompass all lesson hours. In class D, the indices (M) and (A) indicate that the analyses were conducted during the morning

hours (from 9:00 AM to 1:00 PM) and afternoon hours (from 2:00 PM to 6:00 PM), respectively. Table 3 reports the values obtained for the I_{powExp} model (2) during the selected events in the four classes A, B, C, and D used to reproduce the CO_2 concentrations.

	A	B	C	D
$CO2_0$ (ppm)	439,96	395,47	352,79	413,88
K_{pow}^{CO2}	0,0008	0,27	0,242	0,37
K_{exp}^{CO2} (min^{-1})	-0,0001	-0,005	-0,002	0,00
R	0,65	0,90	0,88	0,88
T_d (min)	540	254	780	303

Table 3: Mean parameter values obtained from the I_{powExp} model during the selected days

The PowerExp model demonstrates robust performance in reproducing the temporal CO_2 concentration profiles for classes B, C, and D, with correlation coefficients approximately $R \approx 0.89$. In contrast, type A events yield lower average correlations ($R \approx 0.66$), which can be attributed to the absence of occupants. Under unoccupied conditions, the temporal evolution of CO_2 predominantly reflects dilution and dispersion mechanisms intrinsic to the indoor environment. The parameter K_{pow} exhibits a monotonic linear increase across classes A to D, showing a strong positive correlation with occupant density. Conversely, the parameter K_{exp} demonstrates an inverse relationship with occupancy levels, plausibly linked to occupant-induced air mixing and activity. This results in a transition from quasi-stagnant indoor air conditions in class A to enhanced outdoor air infiltration and dynamic mixing in class D.

4. Conclusions

The I_{powExp} model accurately characterizes the temporal evolution of CO_2 concentrations as a function of the mean student occupancy. The parameters K_{pow} and K_{exp} exhibit statistically significant correlations with the measured number of occupants. This model effectively describes the filling and depletion dynamics driven by student presence within a university classroom environment and provides reliable forecasts of peak CO_2 concentration events.

References

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[3] Persily A, de Jonge L. Carbon dioxide generation rates for building occupants. Indoor Air. 2017.