

2 LENGTH-SCALES FOR INCLINED STABLE BOUNDARY LAYERS

Branko Grisogono & Danijel Belušić

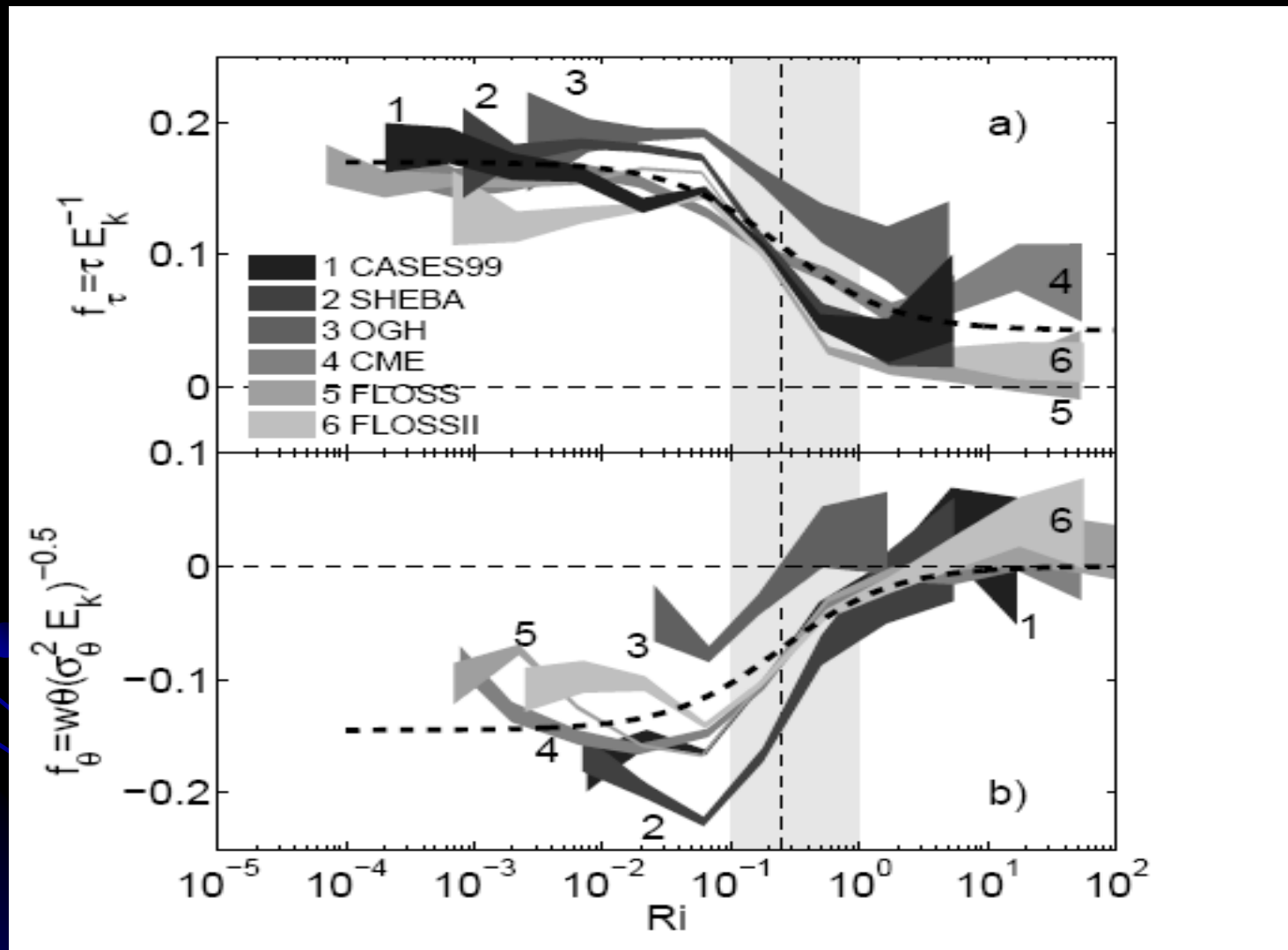
*Dept. of Geophysics, Faculty of Sci., Univ. of Zagreb
Croatia*

TNX to: Amela Jeričević, Lukša Kraljević, Iva Kavčić & Leif Enger

OUTLINE

- 1) Modeled SABL over-diffused; 2) Turbulence for $Ri < 1$
& $Ri > 1$
- MODIFIED PRANDTL MODEL :
- MONIN-Obukhov LENGTH vs. *LLJ* HEIGHT
- ADJUSTING “z-less” MIXING LENGTH FOR *SABL*

- Mauritsen et al. JAS 2007
 ←
→



- *z-less Mix. length modified*

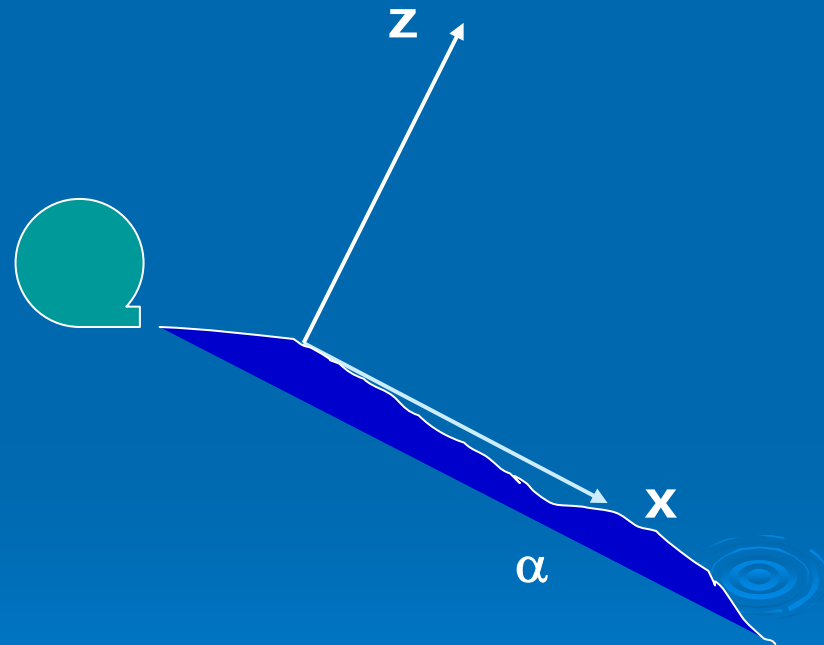
- **PRANDTL MODEL**

- $L_{\text{MONIN-OBUKHOV}}$ *modified*

(if a sfc. layer exists)

PRANDTL MODEL

- Analytical model for the “pure” katabatic flow
- Balance between the negative buoyancy & turbulent diffusion
- $\alpha < 0$



PRANDTL MODEL – *cont'd*

- Simplified N-S eqns.
- Heat (pot. temperature) eqn.
- Assume:
 - **steady-state** [later: semi-stationarity (\mathbf{U} & θ , not \mathbf{V})]
 - **Boussinesq**
 - **hydrostatic balance**
 - **linearity** (no advection) but for finite amplitude perturb.
 - **simple K -theory** [later: $\mathbf{K} = \mathbf{K}(z)$]
 - **$\gamma = d\theta/dz = \text{constant}$, $\theta^{\text{tot}} = \theta + \gamma z$, ($\Theta = \theta_0 + \theta^{\text{tot}}$)**

PRANDTL: CLASSICAL ANALYTICAL MODEL

- Scale analysis: $f\cos(\alpha)V$ small in x -eqn. for U
- Stationarity for U & θ after $t > 1-1.5T$, $T = 2\pi/(N\sin(\alpha))$,

$$U: \quad 0 = \frac{g}{\theta_0} \sin(\alpha)\theta + \text{Pr} \frac{d}{dz} \left(K \frac{dU}{dz} \right)$$

$$\theta: \quad 0 = -\gamma \sin(\alpha)U + \frac{d}{dz} \left(K \frac{d\theta}{dz} \right)$$

MIUU: Modified 'z-less' Mixing length

- *Replace:*

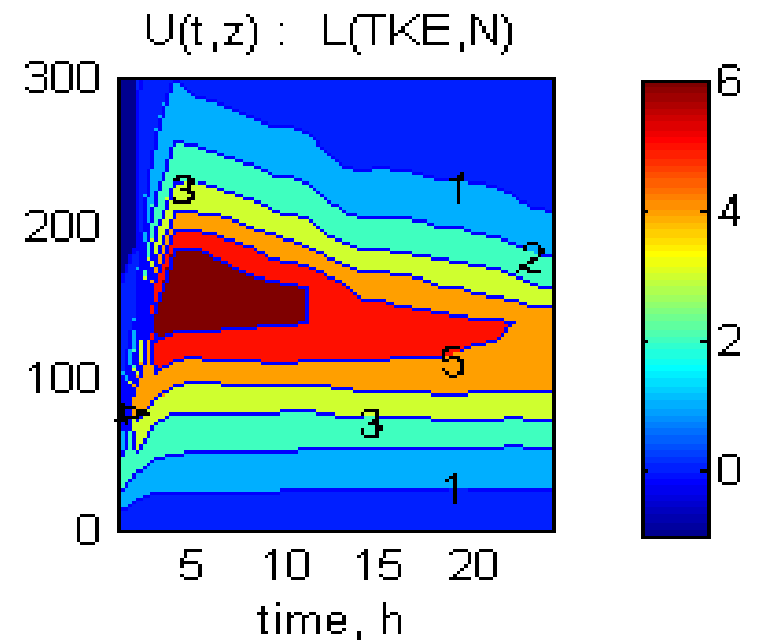
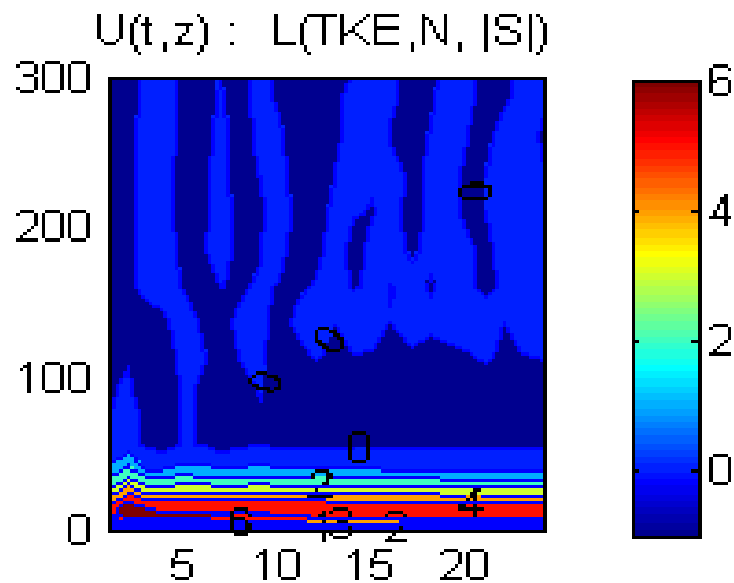
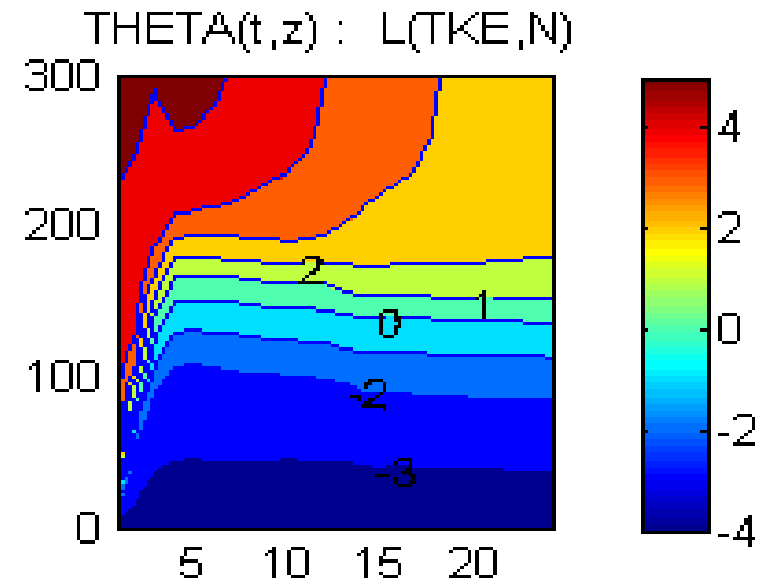
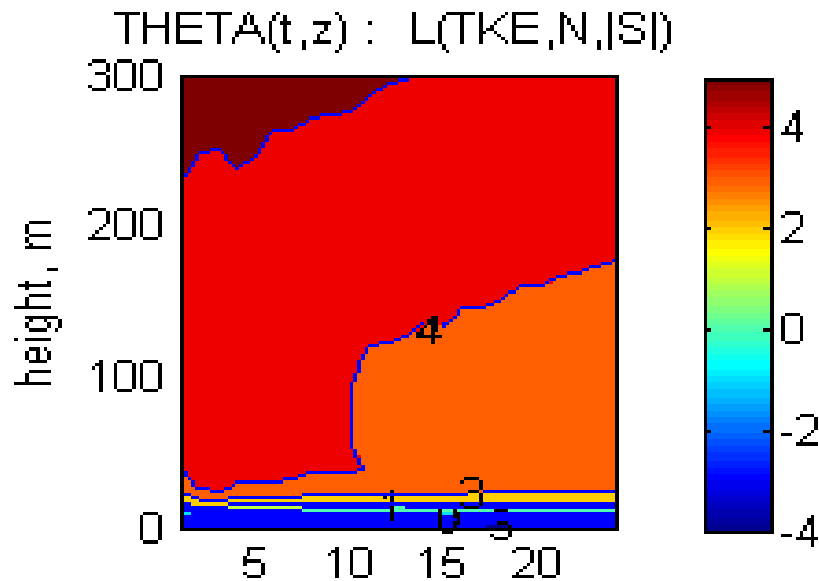
$$l_{STAB} = a \frac{(TKE)^{1/2}}{N}$$

- *By:*

$$l_{STAB} = \min \left[a \frac{(TKE)^{1/2}}{N}, b \frac{(TKE)^{1/2}}{|S|} \right]$$

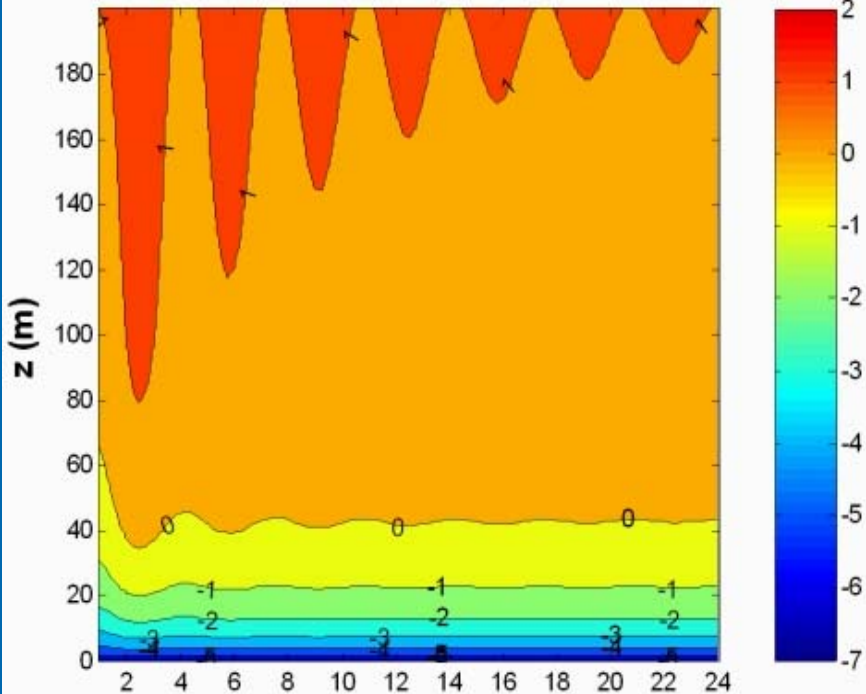
- ... and find 'b' ('a' known, in MIUU model: $a \approx 0.5$)

Same start | **L: Correct VSABL** | **R: Over-Diffusive SABL**



(a)

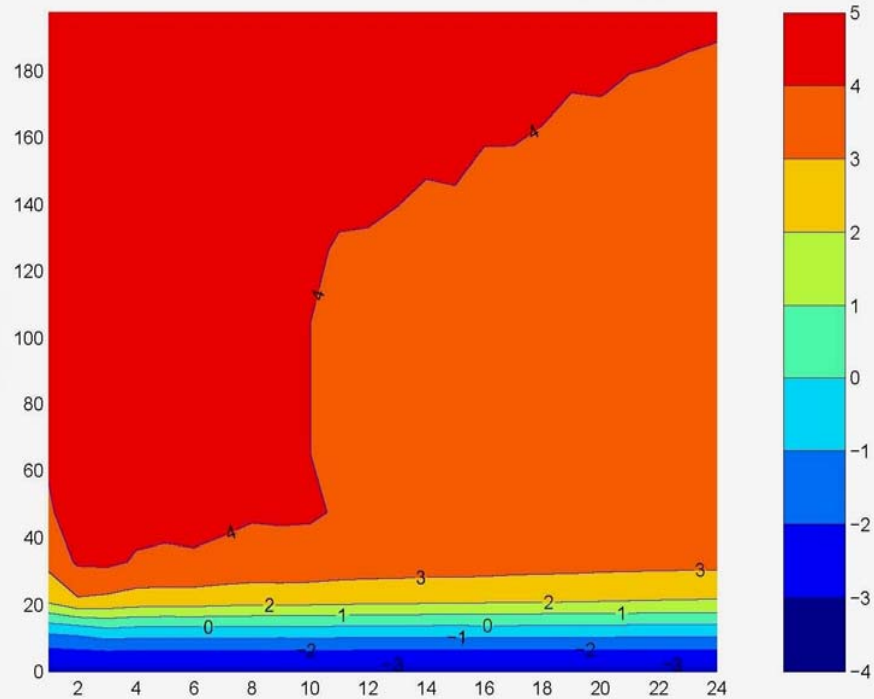
$$\theta_{num}^{tot} (\text{°C})$$



t (h)

(b)

$$\theta_{MIUU}^{tot} (\text{°C})$$



t (h)

(a) simple numerical (*oscil. with T*)

(b) MIUU model

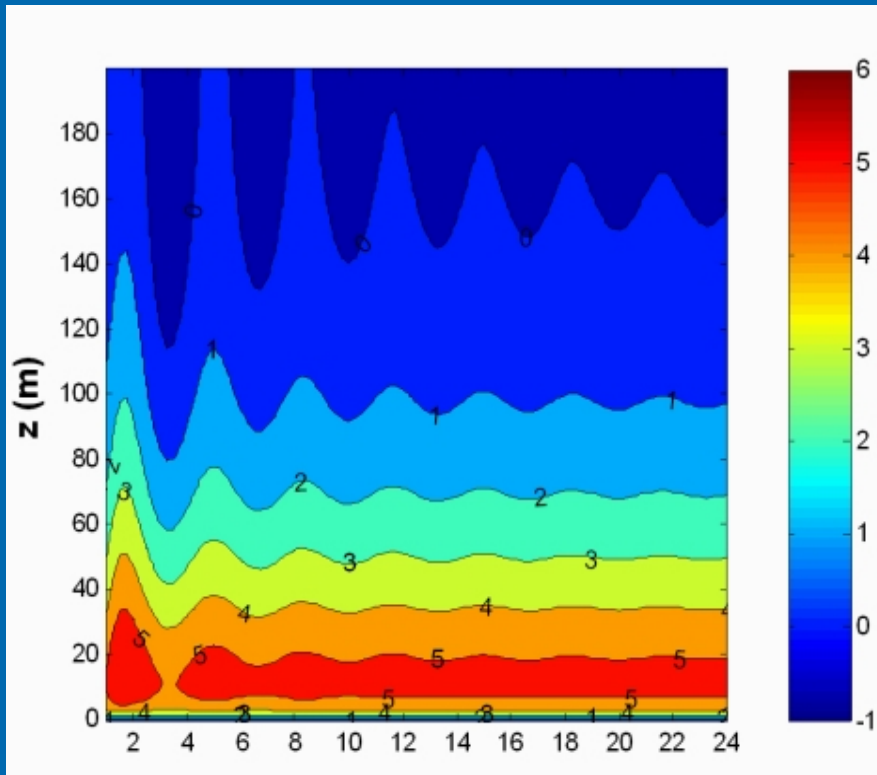
$(f, \alpha, \gamma, Pr, C) = (1 \cdot 10^{-4} \text{ s}^{-1}, -2.2^\circ, 5 \cdot 10^{-3} \text{ Km}^{-1}, 1.1, -6.5 \text{ °C})$

$K(z): h = 200 \text{ m}, K_{max} = 2 \text{ m}^2\text{s}^{-1}$

here: $T = 3.4 \text{ h}$

(a)

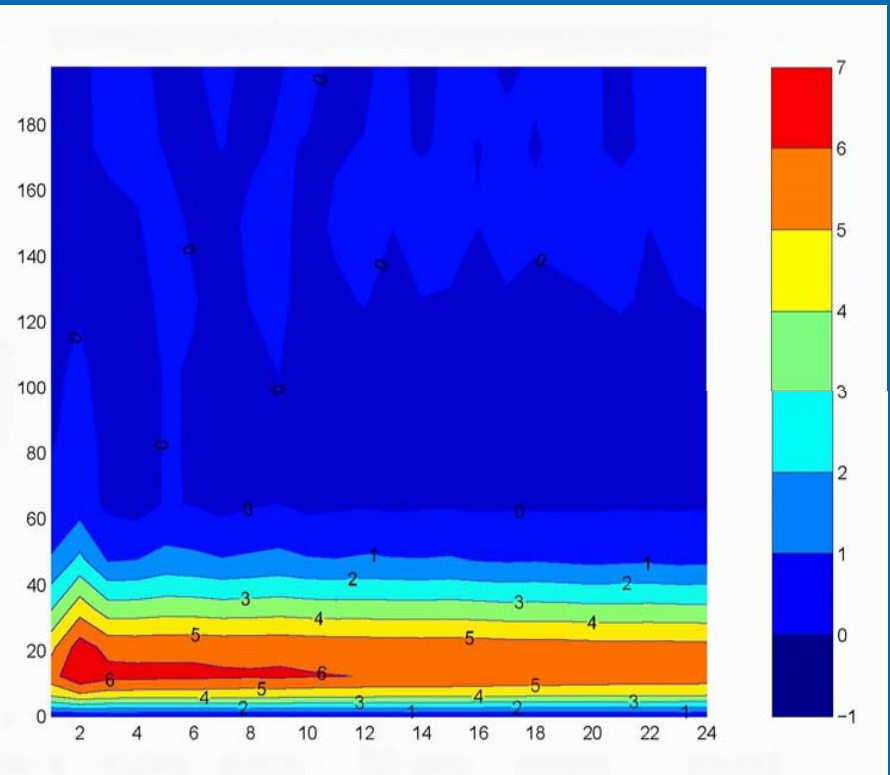
$$U_{num} (ms^{-1})$$



t (h)

(b)

$$U_{MIUU} (ms^{-1})$$



t (h)

(a) simple numerical

(b) MIUU model

The rest as before.

$Max(U_{num}) \approx 5.5 \text{ ms}^{-1}$ & LLJ height $\approx 16 \text{ m}$

Numerical (U, θ) with elevated spurious oscillations decaying in time, $T \approx 3.4h$

Conclusion 1/2: MIXING LENGTH

- (U, θ) reach *steady state* after $t > T$; V_{num} & V_{WKB} diffuse upwards
- Simple numerical, WKB & MIUU MODEL agree for (θ_{tot}, U, V) **only with new Mix. Length**
- USE:
Very **SABL**, $Ri \rightarrow \infty$, \Rightarrow difficult to treat in NWP & climate models (usually over-diffusive) \Leftrightarrow a solution is offered

- MONIN-Obukhov vs. *LLJ* HEIGHT

$$L_{MO} = \frac{-\bar{\theta} u_*^3}{gk w' \theta'}$$

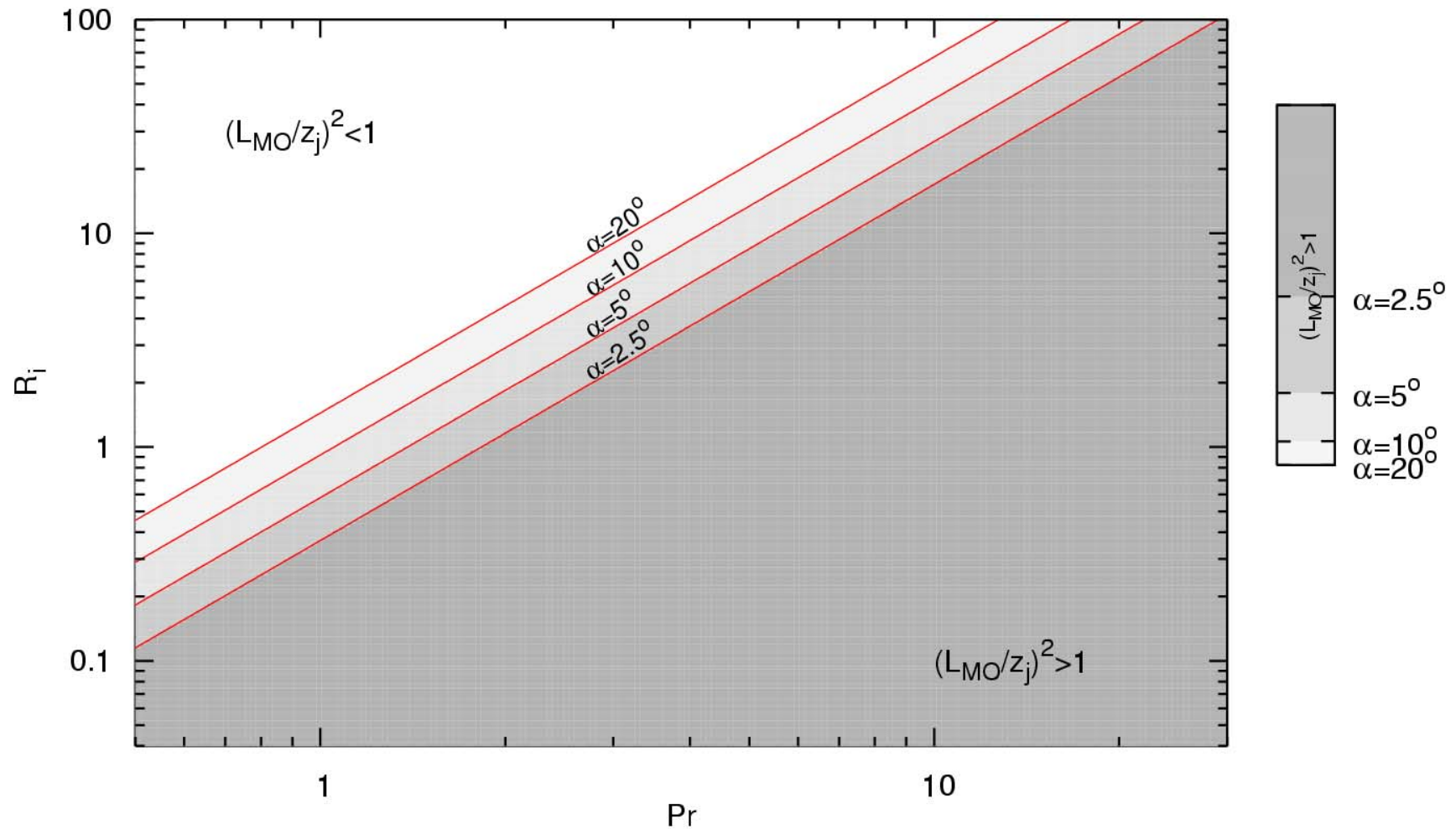
$$z_j = \frac{\pi}{4} \left(\frac{4K^2 \text{Pr}}{N^2 \sin^2(\alpha)} \right)^{1/4}$$

$$\left(\frac{L_{MO}}{z_j} \right)^2 = \frac{8}{(k\pi)^2} |\sin(\alpha)| \left(\frac{\text{Pr}^5}{\text{Ri}^3} \right)^{1/2}$$

- if α small, e.g. < 0.1 rad & $\text{Pr} \approx \text{Ri} \sim 1$, $\Rightarrow L_{MO} < z_j$, \Rightarrow

classical Monin-Obukhov OK; note **N** is external param.

- For $(L_{MO}/z_j) > 1$, L_{MO} is less relevant scale for sfc. fluxes



- **CONCLUSION** 2/2: A modified L_{MO} includes the LLJ

height:

- $L_{mod} = \min(L_{MO}, c z_j)$, $c \sim 0.8$

- Or at least:

$$L_{MOD}^{-1} = aL_{MO}^{-1} + bz_j^{-1}$$

- For NWP & climate models
- It tells where/when classic L_{MO} wrong in **sloped very SABL**

APPENDIX

- If discussion desires, continue ...
- Otherwise, be seated quietly...
- & Let the others continue ...

MODIFIED PRANDTL: [f & $K(z)$]

$$\frac{\partial U}{\partial t} = \frac{g}{\theta_0} \sin(\alpha) \theta + f \cos(\alpha) V + \text{Pr} \frac{\partial}{\partial z} \left(K \frac{\partial U}{\partial z} \right)$$

$$\frac{\partial V}{\partial t} = -f \cos(\alpha) U + \text{Pr} \frac{\partial}{\partial z} \left(K \frac{\partial V}{\partial z} \right)$$

$$\frac{\partial \theta}{\partial t} = -\gamma \sin(\alpha) U + \frac{\partial}{\partial z} \left(K \frac{\partial \theta}{\partial z} \right)$$

B. C.

$$\theta(z=0) = C, \quad U(z=0) = V(z=0) = 0$$

$$\theta(z \rightarrow \infty) = U(z \rightarrow \infty) = V(z \rightarrow \infty) = 0$$

- PRANDTL: WKB SOLUTIONS FOR (θ, U, V) ; $K = K(z)$

$$\theta_{WKB} = C \exp(-\sigma_{WKB}(z)) \cos(\sigma_{WKB}(z)),$$

$$U_{WKB} = \frac{C \sigma_0^2}{\gamma \sin(\alpha)} \exp(-\sigma_{WKB}(z)) \sin(\sigma_{WKB}(z)),$$

$$V_{WKB} \approx \frac{-Cf \cot(\alpha)}{\text{Pr} \gamma} \left[1 - \text{erf}\left(\frac{I(z)}{2\sqrt{t} \text{Pr}}\right) - \exp(-\sigma_{WKB}(z)) \cos(\sigma_{WKB}(z)) \right],$$

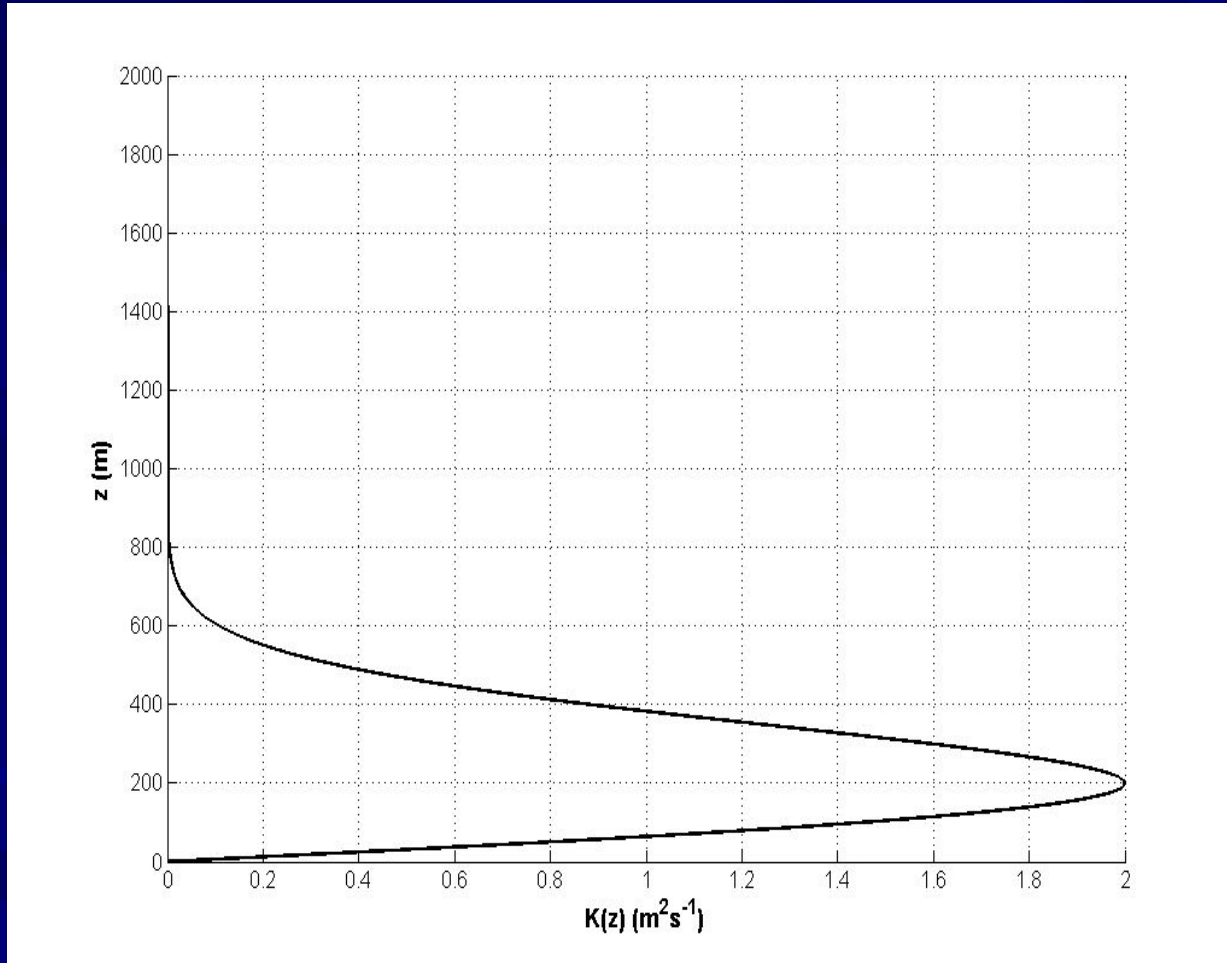
$$\sigma_{WKB}(z) = \frac{\sigma_0}{\sqrt{2}} I(z),$$

$$\sigma_0^4 = \frac{N^2 \text{Pr} \sin^2(\alpha) + f^2 \cos^2(\alpha)}{\text{Pr}^2}$$

$$I(z) = \int_0^z K(z)^{-1/2} dz,$$

- **$K(z)$ profile:**

$$K(z) = K_{\max} \sqrt{e} \frac{z}{h} \exp\left(-\frac{z^2}{2h^2}\right)$$



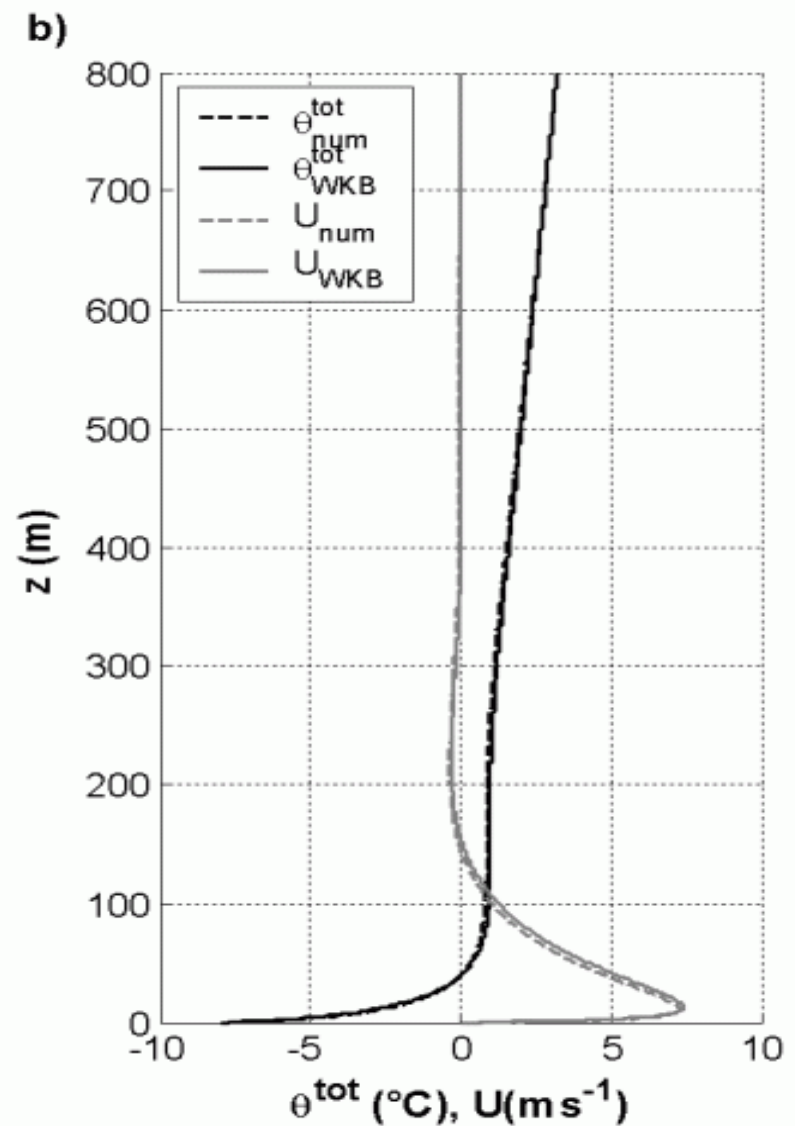
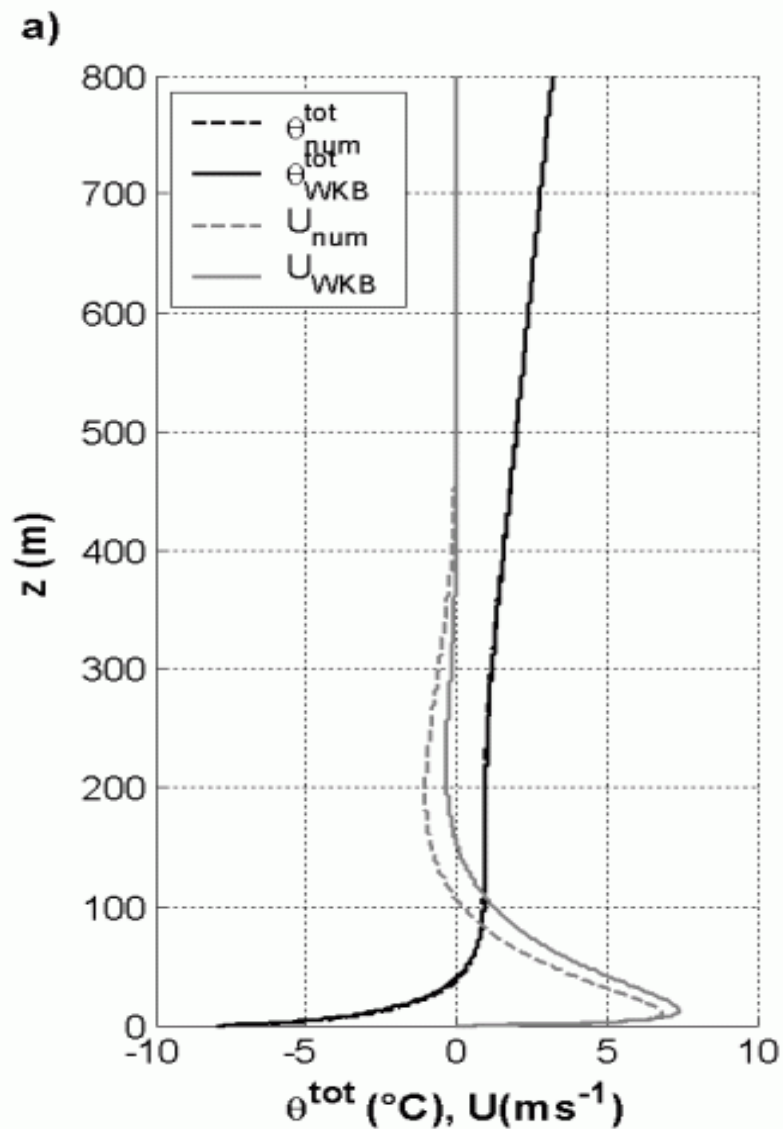
$K(z)$ for numerical &
analytical solutions,
only 2 parameters!

e.g.

$$K_{\max} = 2 \text{ m}^2\text{s}^{-1},$$

$$h = 200 \text{ m},$$

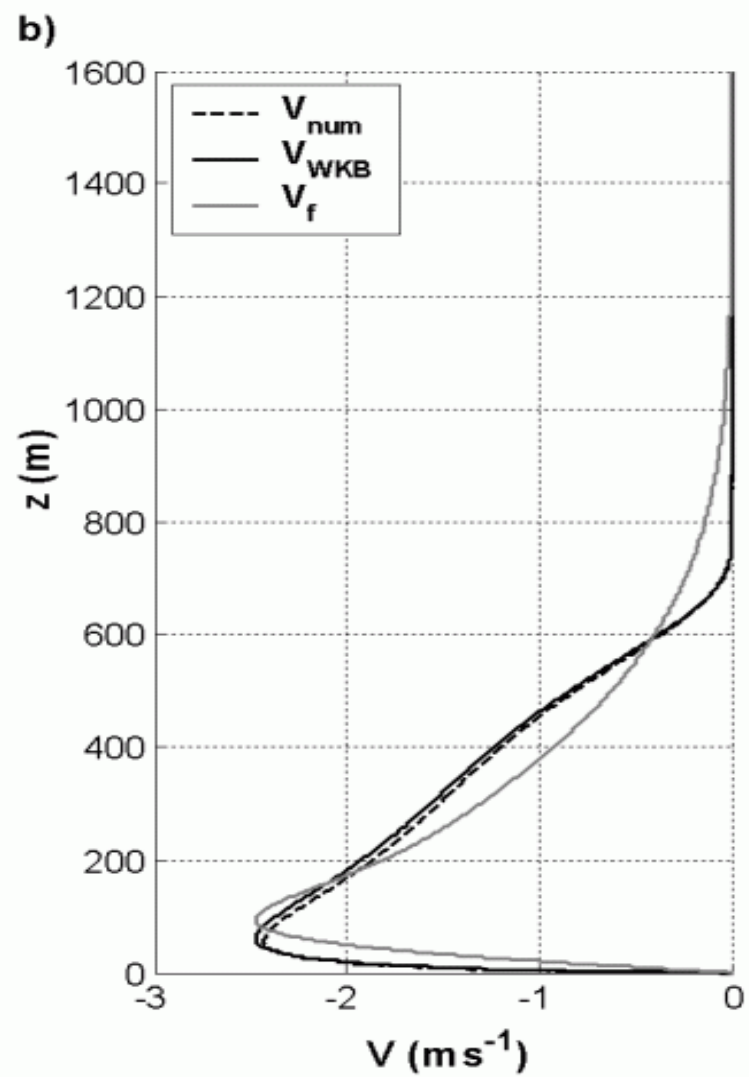
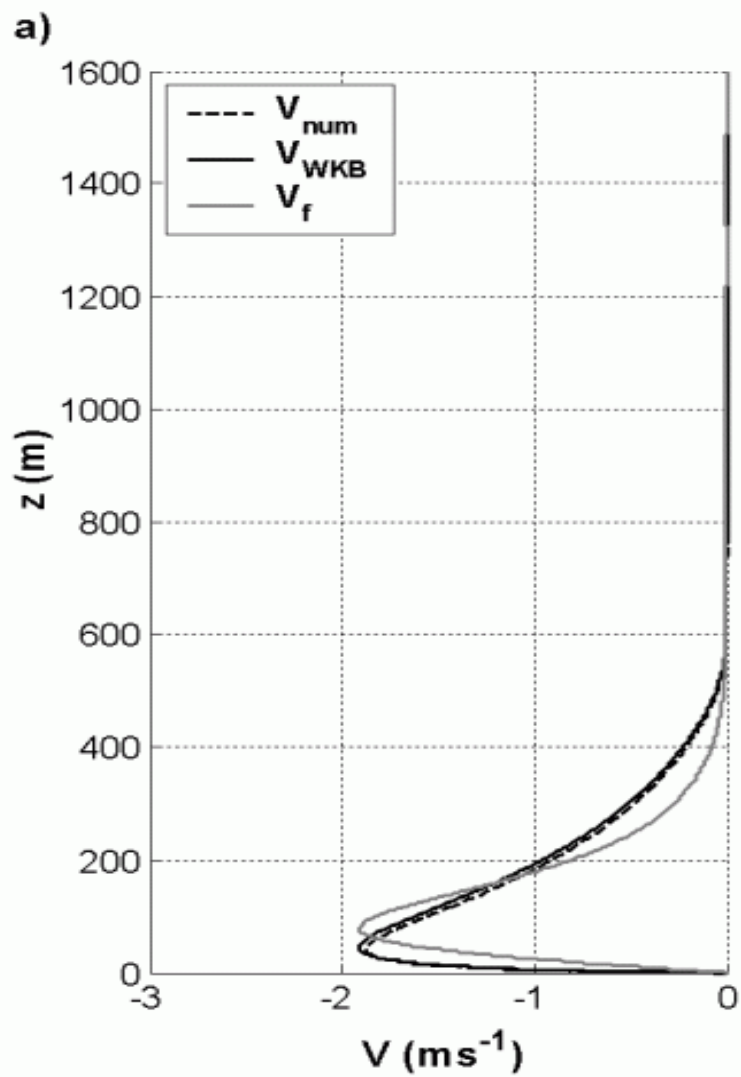
(JAS 2001, QJ2001,...)



a) $t = T$

$(\alpha, \gamma, Pr, C, f) = -4^\circ, 4 \cdot 10^{-3} \text{ Km}^{-1}, 1.1, -8^\circ\text{C}, 1.1 \cdot 10^{-4} \text{ s}^{-1}$

b) $t = 10T, T \approx 2.1 \text{ h}$



(a) $t = T$

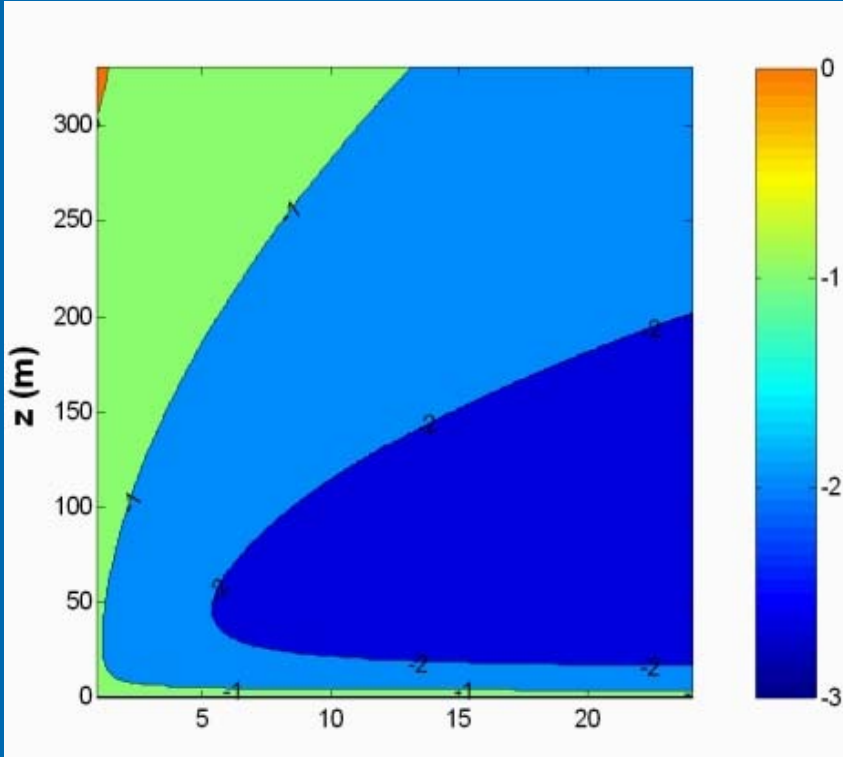
(Stiperski et al. QJRMS 2007; Kavčič & B.G., BLM 2007)

(b) $t = 10T$

V_f uses $K = \text{const} = K_{\text{max}} / 3$

(a)

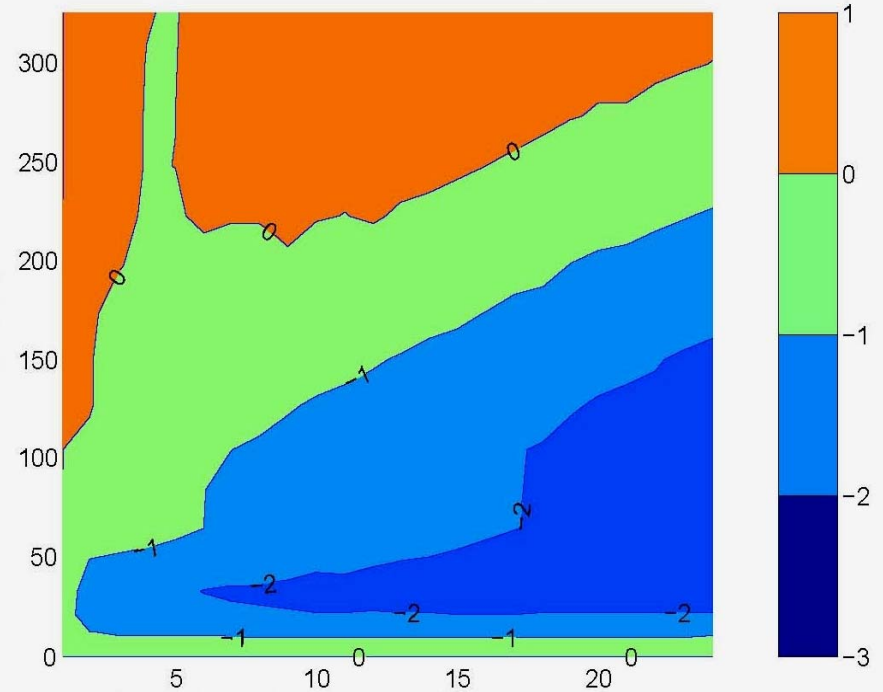
$$V_{WKB} (ms^{-1})$$



t (h)

(b)

$$V_{MIUU} (ms^{-1})$$



t (h)

(a) WKB solution

(b) MIUU model

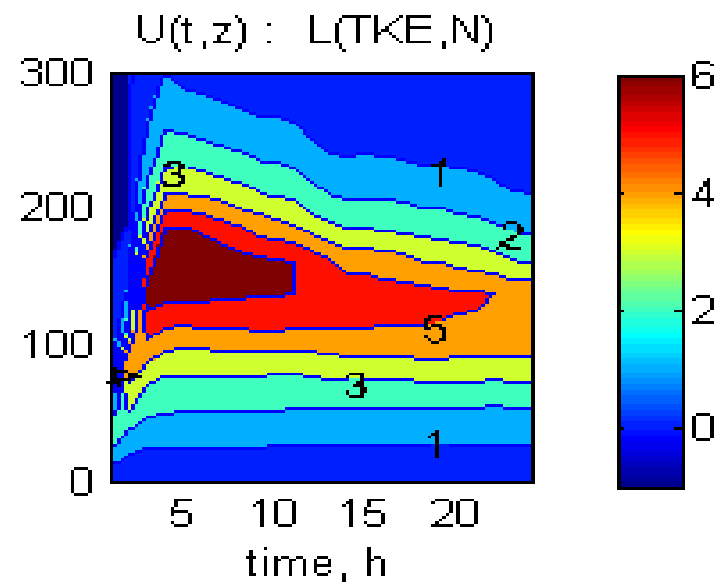
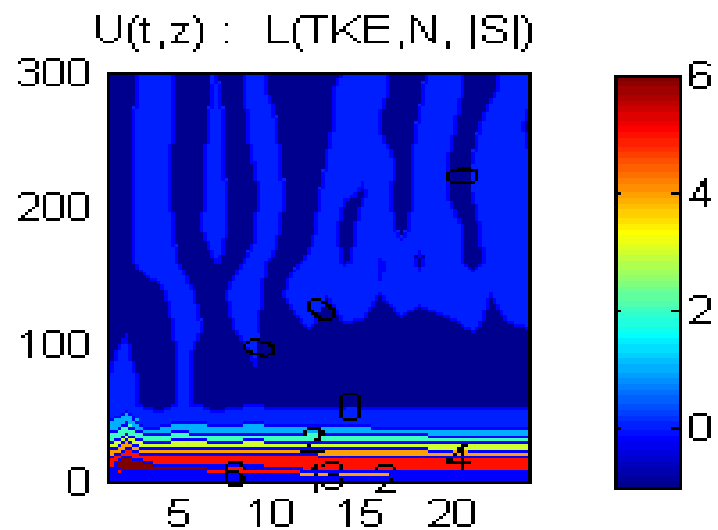
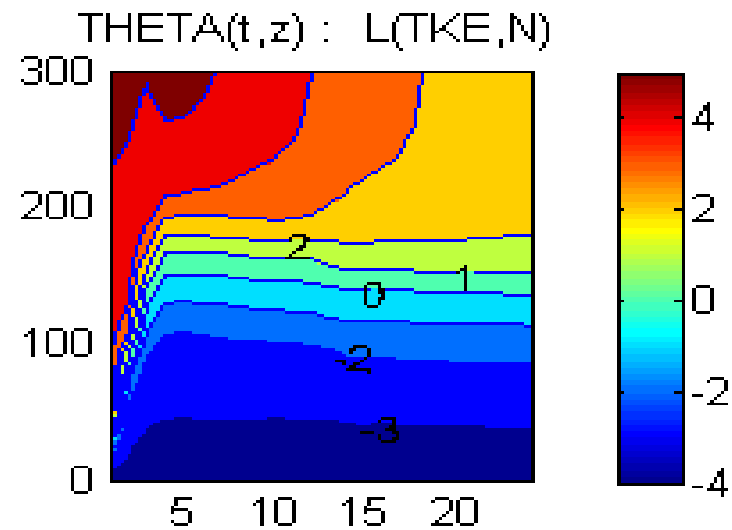
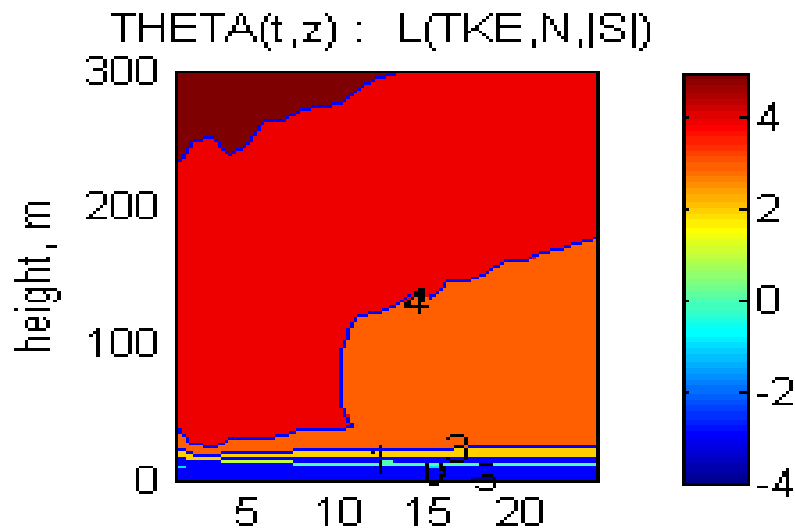
The rest as before. $\text{Min}(V_{WKB}) \geq -2.75 \text{ ms}^{-1}$

WKB sol. has no spurious (low energy) oscillations

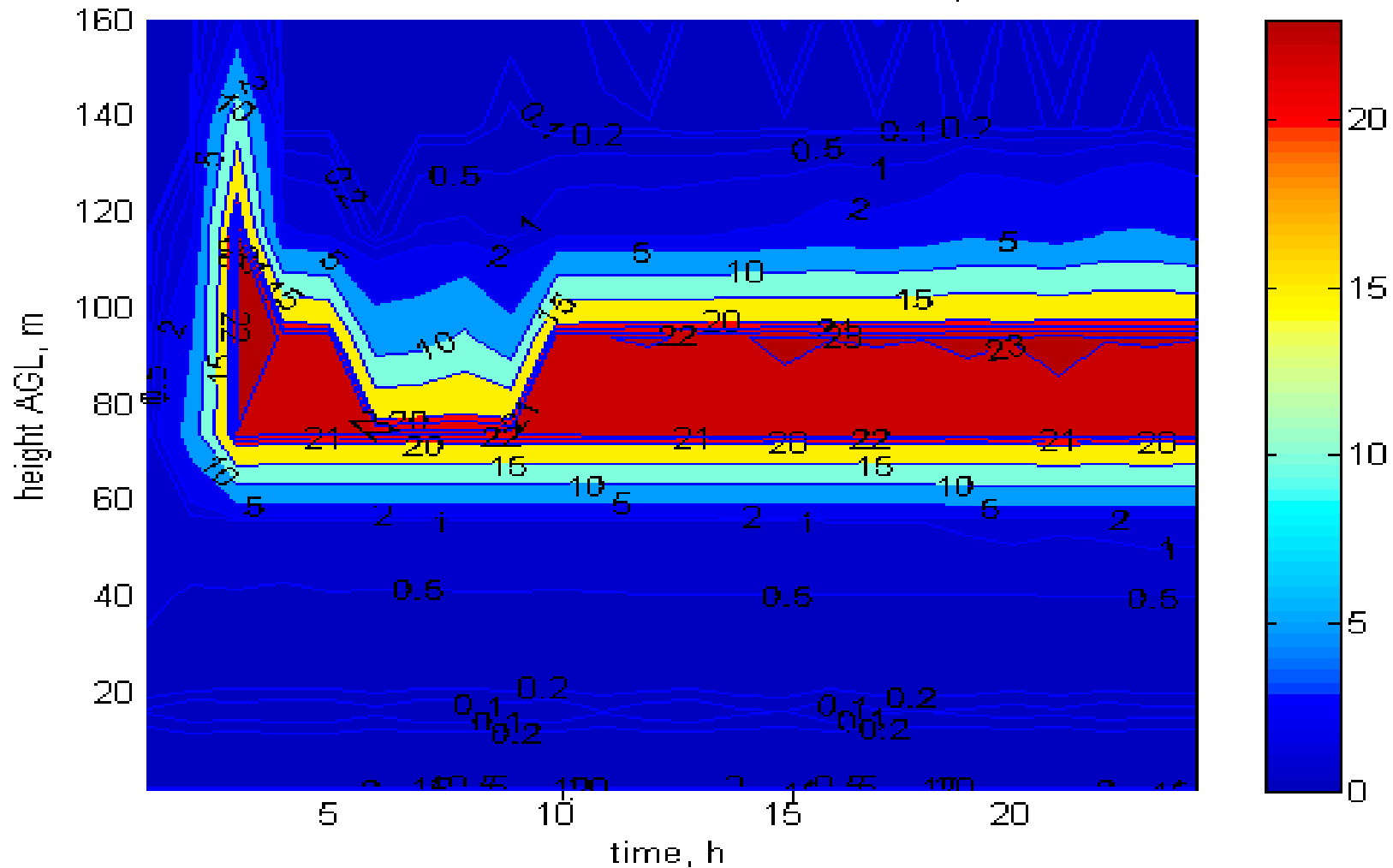
MIUU MODEL & Simulation Parameters

- nonlinear, hydrostatic, f-plane
 - 2.5-turbulence closure level
 - 5 progn. eqns: U, V, θ, q, TKE (e.g. QJRMS 2004)
-
- $\Delta x \approx 1.9 \text{ km} = \text{const}$, $\alpha = -2.2^\circ$, $C \approx -6.5^\circ\text{C}$
 - $211 \times 7 \times 201$ gridpoints ($y = \text{const}$), total(nk)=402
 - $\Delta t = 13 \text{ s}$, $z_{TOP} = 5.6 \text{ km}$ (“sponge” for $z \geq 4.4 \text{ km}$)
 - $1 \text{ m} < \Delta z < 30 \text{ m}$, “staggered, terrain influenced”
 - Initialisation: $U \ \& \ V \approx 0$, $\gamma = 5 \cdot 10^{-3} \text{ Km}^{-1}$

$$L_{STAB} = \min[a(TKE)^{1/2}/N, a/2(TKE)^{1/2}/|S|], \quad a \approx 0.5$$



MIUU model TURB. MIXING LENGTH, m



$$L_{STAB} = \min[a(TKE)^{1/2}/N, a/2(TKE)^{1/2}/|S|], \quad a \approx 0.5$$