CONCENTRATION FLUCTUATIONS INSIDE A PLANT CANOPY.

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INTRODUCTION

The knowledge of the concentration probability density function (pdf) is essential in many practical applications. In order to predict its moments in canopy generated turbulence a Lagrangian fluctuating plume model has been developed. While the behaviour of the mean concentration field of a non-reactive tracer inside a canopy is relatively well understood high concentration statistics and in particular concentration fluctuations predictions are still matter of discussion.

Starting from the ideas of Gifford (1959) and Yee and Wilson (2000) analitical models for predicting concentration moments of order higher than the first for stationary release of passive tracers in homogeneous turbulence, Luhar et al. (2000) proposed a Lagrangian stochastic model for the evolution of the barycentre of the plume of contaminants able to take in to account the vertical inhomogeneity and the skewness of the turbulent velocity of a convective boundary layer. the concentration field is evaluated parametrizing the dispersion of the plume relative to the instantaneous centre of mass of the cloud. Following these works Franzese (2003) developed a one-dimensional fluctuating plume model able to describe all the moments of the concentration field in a vertical inhomogeneous flow.

In this work we developed a fluctuating plume model based on Franzese (2003) and Luhar et al. (2000) modes for turbulent flows generated by the presence of a vegetal canopy and in particular we simulated the experiment of scalar dispersion within a model plant canopy performed by Raupach et al. (1986). The moments of turbulence are filtered in order to consider only the proper portion of the turbulent kinetic energy and to derive the equations describing the istantaneous movement of the plume barycenter, thus being able to determine the plume centroid PDF. The concentration field is subsequently calculated parametrizing the relative dispersions around the cloud centre of mass using a gamma distribution. The mean concentration and standard deviation evaluated by the model are then compared with Legg et al. [1986] data. In order to correct an under-estimation of the measured mean concentrations close to the ground, a correction suggested by Dosio and de Arellano [2006] is applied.

CONCENTRATION STATISTICS

Following Gifford (1959), we assume that the concentration pdf of a passive tracer can be written as:

$$p(c;x,z) = \int p_{cr}(c|x,z,z_m)p_m(x,z_m)dz_m \qquad (1)$$

where $p(c;x,z)$ is the concentration pdf in the fixed system, $p_{cr}(c|x,z,z_m)$ is the concentration pdf in a reference frame whose origin is located at the plume barycentre position (z_m) and $p_m(x,z_m)$ is the z_m pdf at a given distance x from the source. From now on we would consider a two dimensional space where turbulence is homogeneous in the along wind direction x and inhomogeneous in the vertical direction z.

Dividing dispersion in two components: the meandering of the instantaneous plume and the relative diffusion of the cloud around its barycentre alllows us to evaluate all the concentration moments as:

$$\left\langle c^{n}(x,z)\right\rangle = \int \left\langle c_r^{n}\right\rangle p_m(x,z_m)dz_m \qquad (2)$$

where

$$\left\langle c_r^{n}(x,z,z_m)\right\rangle = \int_0^{\infty} c^{n} p_{cr}(c|x,z,z_m)dc \qquad (3)$$

Equation (2) shows that the concentration statistics can be evaluated integrating the concentration statistics relative to the cloud centroid on the centroid pdf. In the fluctuating plume approach $p_m(x,z_m)$ is evaluated through a Lagrangian stochastic model that simulates the centroid trajectories in a fixed coordinate system while $\left\langle c_r^{n}(x,z,z_m)\right\rangle$ is parametrized.

ENERGY SCALES

Equation (1) suggests that, while the motion of fluid particles is governed by the entire spectrum of the turbulent kinetic energy (tke), the motion of the barycentre is only determined by eddies whose wavelenghts are larger than the plume characteristic scale, the remaining portion of energy is responsible of the internal mixing within the cloud. Thus the vertical comoponent of the turbulent kinetic energy can be thought as (Franzese, 2003):

$$s_w^2 = s_m^2 + s_r^2 \qquad (4)$$

where s_w^2 is the total vertical tke, s_m^2 is the energy reponsible of the centroid meandering and s_r^2 is related to the in-plume fluctuations. Using a simple similarity scaling we can write s_m^2 as a function of s_w^2:

$$s_m^2 = s_w^2\left[1-\left(\frac{d^2}{d^2+H^2}\right)^{1/3}\right] \qquad (5)$$

with H the boudary layer height and $d = d(t)$ the instantaneous size of the cloud. The square parenthesis in equation (5) is a time-dependent low-pass filter that extract the energy related to the cloud centroid motion.

FLUCTUATING PLUME MODEL

The motion of the plume barycentre can be simulated by the following stochastic differential equations (Franzese, 2003):

$$\begin{aligned} dx_m &= U(z_m)dt \\ dw_m &= a_m(t,w_m,z_m)dt + b_m(t,z_m)dW(t) \qquad (6) \\ dz_m &= w_m dt \end{aligned}$$

where $dW(t)$ represents the increment of a Wiener process with zero mean and dt variance. As can be seen from the first of equations (6) the meandering along the x component is ignored because assumed neglegible in respect of the veritcal one.

$b_m = \sqrt{C_0 e}$ is the diffusion coefficient (Thomson, 1987), where $e = \dfrac{2s_w^2}{C_0 T_L}$ is the dissipation of the turbulent kinetic energy, C_0 is a constant (we chose $C_0 = 2$ in our simulations) and T_L is the Lagrangian time scale.

The acceleration term a_m is derived from the Fokker-Planck equation associated with the second of equations (6):

$$\frac{\partial P_m}{\partial t}+w\frac{\partial P_m}{\partial z_m}=-\frac{\partial\left[a_m(w_m,z_m,t)P_m\right]}{\partial w_m}+\frac{\sigma_m^2}{T_m}\frac{\partial^2 P_m}{\partial w_m^2}\qquad(7)$$

where the acceleration term is considered quadratic in w_m (Franzese, 2003):

$$a_m(w_m,z_m,t)=\alpha_m(z_m,t)w_m^2+\beta_m(z_m,t)w_m+\gamma_m(z_m,t)\qquad(8)$$

α_m, β_m and γ_m are determined multiplying equation (7) for powers of w_m, integrating it over w_m and then solving the obtained system of equations.
Hence:

$$\alpha_m=\frac{(1/3)\left(\partial\langle w_m^3\rangle/\partial t+\partial\langle w_m^4\rangle/\partial z_m\right)}{\langle w_m^4\rangle-\langle w_m^3\rangle^2/\langle w_m^2\rangle-\langle w_m^2\rangle^2}$$

$$\beta_m=\frac{1}{2\langle w_m^2\rangle}\left[\frac{\partial\langle w_m^2\rangle}{\partial t}+\frac{\partial\langle w_m^3\rangle}{\partial z_m}-2\langle w_m^3\rangle\alpha_m\right]-\frac{1}{T_m}\qquad(9)$$

$$\gamma_m=\frac{\partial\langle w_m^2\rangle}{\partial z_m}-\langle w_m^2\rangle\alpha_m$$

To obtain an explicit form for α_m, β_m and γ_m and so for a_m it is necessary to know the quantities $\langle w_m^2\rangle, \langle w_m^3\rangle$ and $\langle w_m^4\rangle$. In order to simulate Raupach et al. (1986) experiment we applied the energy filter (5) to their measured moments and substituted the results in equations (9).

RELATIVE CONCENTRATION PDF

Once obtained the vertical pdf of the barycentre position, equation (2) allow us to evaluate the concentration field in the fixed coordinates system. Nevertheless to integrate equation (2) it is necessary to have an explicit form for all the moments of the relative concentration $\langle c_r^n\rangle$.

Assuming (Luhar et al., 2000, Dosio and De Arellano, 2006) that the concentration pdf in the reference frame centred in the barycentre of the cloud can be written using a Gamma pdf as:

$$p_{cr}(c|x,z,z_m)=\frac{\lambda^\lambda}{\langle c_r\rangle\Gamma(\lambda)}\left(\frac{c}{\langle c_r\rangle}\right)^{\lambda-1}e^{-\frac{\lambda c}{\langle c_r\rangle}}\qquad(10)$$

and that the instantaneous relative mean concentration in the same frame is (Franzese, 2003):

$$\langle c_r\rangle=\frac{Q}{U(z_m)}p_{zr}(x,z,z_m)\qquad(11)$$

the n-moment of the concentration pdf can be evaluated as (Luhar et al., 2000):

$$\langle c^n(x,z)\rangle=\frac{1}{\lambda^n}\frac{\Gamma(n+\lambda)}{\Gamma(n)}\left(\frac{Q}{U}\right)^n\int_0^H p_{zr}^n(x,z,z_m)p_m(x,z_m)dz_m\qquad(12)$$

where Γ is the Gamma function, Q is the release of material per unit of time, U is the horizontal mean velocity profile, $\lambda=i_{cr}^{-2}$, with i_{cr} intensity of the relative concentration fluctuations. p_{zr} is the vertical pdf of mean particle position relative to z_m and its expression is crucial to determine the concentration pdf. Franzese (2003) adopted a Gaussian pdf, but in this way the distribution of $\langle c_r\rangle$ does not include additional skewness in the vertical direction, a part from the one contained in the centroid pdf and the one generated by reflections at the boundaries, hence, as noticed by Dosio and de Arellano (2006), it under estimates the average mean concentration. Following Luhar et al. (2000) and Dosio and de Arellano (2006) we incorporated the skewness in p_{zr}:

$$p_{zr}(x,z,z_m)=\sum_{j=1}^{2}\sum_{n=-N}^{N}\frac{a_j}{\sqrt{2p}\,s_j^2}\left[e^{-\frac{(z-z_m+2nzH-\bar{z}_j)^2}{2s_j^2}}+e^{-\frac{(-z-z_m+2nzH-\bar{z}_j)^2}{2s_j^2}}\right] \quad (13)$$

where a_j, s_j, $\bar{z}_j$ are parameters depending on the difference between the skewness of the fluctuations of z and z_m.

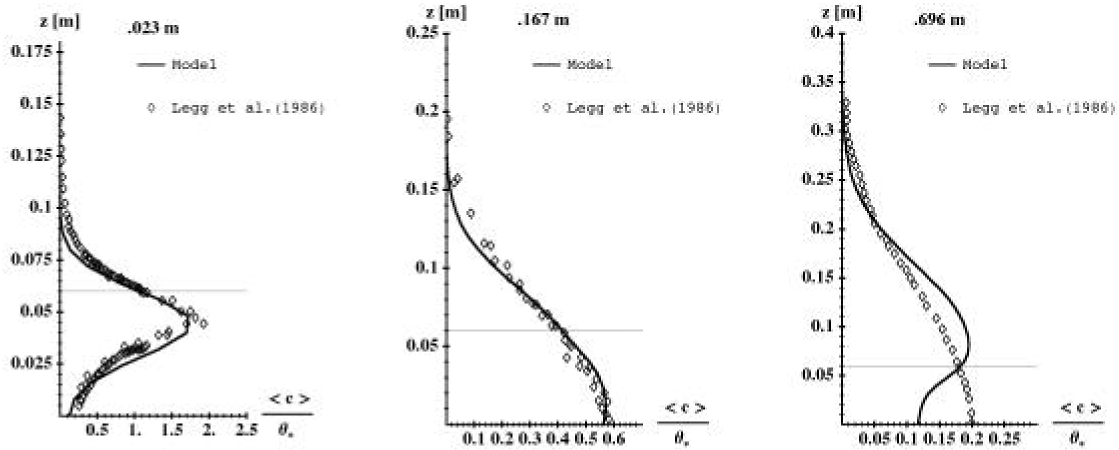

Fig. 1; Mean concentration normalized with the concentration scale q_* (Legg et al., 1986). The horizontal line represents the canopy height. The number at the top of each figure is the distance from the source.

RESULTS AND CONCLUSIONS

Figure 1 shows the absolute mean concentration profiles at three distances from the source. The solid line is the model output, while the diamonds are Legg et al. (1986) measured data. The model presents a very good agreement close to the source, while it shows an underestimation inside the canopy in the far field, this underestimation has not to be connected with an underestimation of the relative concentration skewness, but it is connected with the rising of the plume after the reflection on the ground. Cassiani et al. (2007) model better fits the data far from the source, but it underestimates the mean concentration at the middle distance.

Figure 2 shows the normalized concentration fluctuations at three distances from the source. Close to the source seems that, though Legg et al. (1986) do not entirely plot their data, the model underestimates the measured data, but this result is similar to the one found by Cassiani et al (2007). In the second grapchic of figure 2 the model overestimates the measured data close to the ground. The introduction of a skewed pdf (Luhar et al. 2000, Dosio and de Arellano 2006) increases the value of $\langle c \rangle$ close to the ground but it enlarges the concentration fluctuations when the plume barycentre is very close to the bottom of the boundary. As a matter of fact when the plume barycentre rises (third image in figure 2), the concentration fluctuations predicted by the model better fits the measured data.

The fluctuating plume model, although very simple in its formulation, showed to be a promising approach to simulate relative dispersion in not idealized situations. It avoids the need to assume an analytical (or numerical) form for the velocity PDF and allows the direct use of measured turbulent moments up to the fourth order, therefore it naturally takes into account the inhomogeneity of the turbulent velocity field and, through the energy filter function, its non-stationarity.

It also has to be stressed that although we presented only the mean concentration and the concentration fluctuations, the fluctuating model is able to evaluate the higher order moments of the concentration pdf.

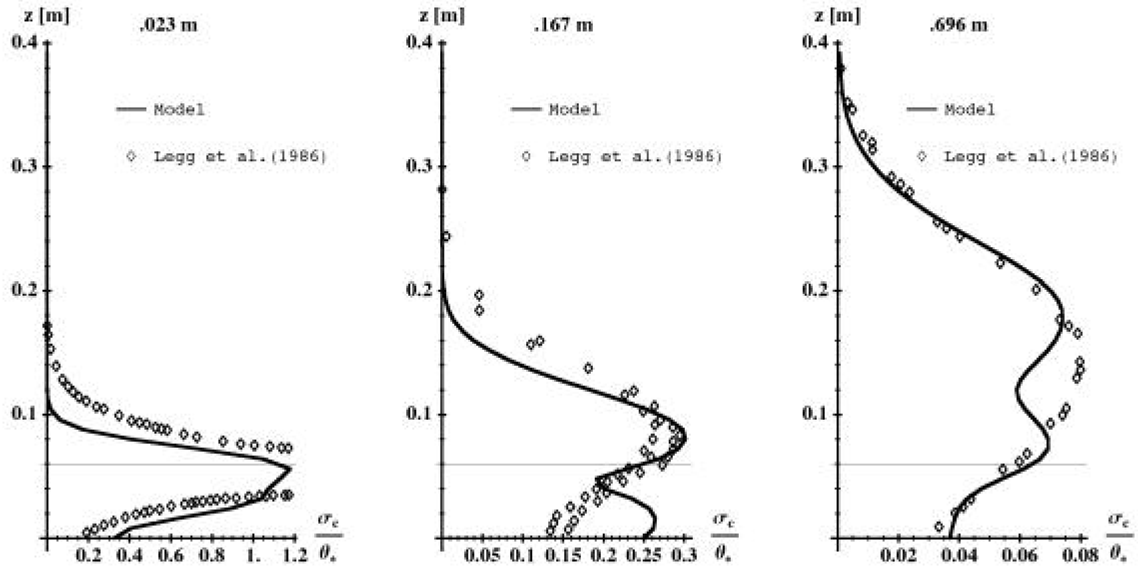

Fig. 2; Concentration standard deviation normalized with the concentration scale q_* (Legg et al., 1986). The horizontal line represents the canopy height. The number at the top of each figure is the distance from the source.

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