PLUME DESCRIPTORS FROM AN ANALYTICAL SOLUTION OF THE ADVECTION-DIFFUSION EQUATION

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INTRODUCTION

The vertical distribution for a point source plume is studied through the statistical descriptors derived from the analytical solution of the advection-diffusion equation. Traditionally operative modelling for dispersion has been performed adopting a Gaussian approach taking in account atmospheric turbulence assuming simple formulae for concentration distribution, where the parameterization depend simply on downwind distance as well as the meteorological state of ABL (Arya, 1999). Regarding the vertical dispersion the scheme performs adequately for short horizontal distances and for near ground sources only. Within this scheme the low source condition has the effect of the crude approximation of infinite height of the ABL. The Gaussian approach turns out to overestimate the centroid $\bar{z}$ and the variance σ_z^2 when the horizontal distance from the source approaches to the length scale of the real ABL. On the other hand the predicted ground level concentration, regardless the ABL scenario, underestimate the experimental data (Irwin, 1983). In fact, although the ABL is assumed to have infinite height, its real vertical limit affects the behaviour of all evaluated quantities. Non-Gaussian approaches are proved to be more reliable, using more adequate parameterizations of the ABL dynamics (Lin and Hildemann, 1996; Brown and Arya; 1997, Tirabassi, 2003).

One of the central equations to describe the evolution of pollutants in the ABL is the Advection-Diffusion Equation (ADE), which is in most cases solved numerically. In the following is reported a study of the two-dimensional steady concentration distribution and its vertical symmetries obtained using the analytical approach GILTT (General Integral Laplace Transform Technique; Wortmann et al, 2005; Moreira et al., 2005). The analytical solution does not present restriction on the ABL parameterization, it is exact except for a round-off error.

THE GILTT SOLUTION

The stationary ADE solved in two-spatial dimensions is the following:

$$u(z)\frac{\partial C(x,z)}{\partial x} = \frac{\partial}{\partial z}\left(K_z(z)\frac{\partial C(x,z)}{\partial z}\right); \quad (1)$$

where *C(x,z)* is the *y* cross-wind integrated steady state concentration. Boundary conditions impose zero flux at the ground (z = 0) and at the ABL height ($z = h$); the emission source is assumed to be point-like and placed at a height h_s above the ground:

$$u(z)c(x,z) = Q\delta(z-h_s)\,; \text{ at } x = 0.$$

Here Q is the pollutant emission rate, *u(z)* is the wind speed in the *x* direction and $K_z(z)$ is the vertical eddy diffusivity. The analytical method GILTT method consists in apllying a series expansion to *C(x,z):*

$$C(x,z) = \sum_{i=0}^{\infty} \bar{c}_i(x)\psi_i(z)\,,$$

where $\bar{c}_i(x)$ and $y_i(z)$ are the solutions of the transformed equation and Sturm-Liouville problem respectively. The resulting transformed equation is then solved, analytically, by applying the Laplace Transform. The infinite series can be truncated when the convergence is limited to a prefixed limit.

THE VERTICAL PLUME DESCRIPTORS

The statistical moments for the vertical distribution are defined as

$$m_1 = \frac{\int_0^h zC(x,z)dz}{\int_0^h C(x,z)dz}, \quad (2)$$

for the first moment, and the higher order as

$$m_m = \frac{\int_0^h (z-m_1)^m C(x,z)dz}{\int_0^h C(x,z)dz}; \qquad m = 2,3,4. \quad (3)$$

The first two moments represent respectively the centroid and the variance ($\bar{z} = m_1$, $s_z^2 = m_2$), and the remaining moments are used to define the dimensionless skewness *Sk* and the kurtosis *Ku* respectively ($Sk = m_3/s^3$, $Ku = m_4/s^4$). Finally the analytical expressions are:

$$\bar{z} = \frac{h}{\bar{c}_0(x)p^2}\sum_{i=1}^{N}\frac{\bar{c}_i(x)}{i^2}\left(\cos ip - 1\right) + \frac{h}{2}, \quad (4)$$

$$s_z^2 = \frac{2h^2}{\bar{c}_0(x)p^2}\sum_{i=1}^{N}\frac{\bar{c}_i(x)}{i^2}\cos ip + \frac{h^2}{3} - \bar{z}^2, \quad (5)$$

$$Sk = \frac{3}{\bar{c}_0(x)p^2}\left(\frac{h}{s_z}\right)^3\sum_{i=1}^{N}\frac{\bar{c}_i(x)}{i^2}\left[\left(1-\frac{2}{i^2p^2}\right)\cos(ip) + \frac{2}{i^2p^2}\right] + \frac{1}{4}\left(\frac{h}{s_z}\right)^3 - \frac{3\bar{z}}{s_z} - \left(\frac{\bar{z}}{s_z}\right)^3, \quad (6)$$

$$Ku = \frac{4}{\bar{c}_0(x)p^2}\left(\frac{h}{s_z}\right)^4\sum_{i=1}^{N}\frac{\bar{c}_i(x)}{i^2}\left(1-\frac{6}{i^2p^2}\right)\cos(ip) + \frac{1}{5}\left(\frac{h}{s_z}\right)^4 - \frac{4\bar{z}Sk}{s_z} - 6\left(\frac{\bar{z}}{s_z}\right)^2 - \left(\frac{\bar{z}}{s_z}\right)^4. \quad (7)$$

It is worth to remind that the second moment is often evaluated in respect of the source height h_s, then substituting h_s in place of $\bar{z}$ in the definition (3) and setting $m = 2$, we get

$$s_s^2 = \frac{2h^2}{\bar{c}_0(x)p^2}\sum_{i=1}^{N}\frac{\bar{c}_i(x)}{i^2}\cos ip + \frac{h^2}{3} - 2h_s\bar{z} + h_s^2. \quad (8)$$

In the following results both expressions for variance will be used.

THE ABL PARAMETERIZATION

In atmospheric diffusion problems the choice of a turbulent parameterization represents a fundamental decision for the pollutants dispersion modelling. The reliability of each model strongly depends on the way turbulent parameters are determined and related to the current understanding of the ABL. We adopt the parameterizations suggested in Degrazia *et al.* (2000). In terms of the convective scaling parameters the vertical eddy diffusivity can be formulated as

$$K_z = 0.22 w_* h\left(\frac{z}{h}\right)^{1/3}\left(1-\frac{z}{h}\right)^{1/3}\left[1-\exp\left(-4\frac{z}{h}\right) - 0.0003\exp\left(8\frac{z}{h}\right)\right], \quad (9)$$

and for stable conditions as

$$K_z = \frac{0.3(1 - z/h)u_* z}{1 + 3.7\, z/\Lambda}, \quad (10)$$

where $\Lambda = L(1 - z/h)^{5/4}$, L is the Monin-Obukhov length, u_* and w_* are the velocity scales for the horizontal friction and vertical convection respectively. The wind speed profile adopted follows the power law expressed as (Panofsky and Dutton, 1988):

$$\frac{\bar{u}_z}{\bar{u}_1} = \left(\frac{z}{z_1}\right)^n, \quad (11)$$

where $\bar{u}_z$ and $\bar{u}_1$ are the mean wind velocity at the heights z and z_1, while n is an exponent related to turbulence. In fact this empirical wind profile matches well similarity profile in the Surface Layer, and on the contrary is valid in all the ABL. The exponent n depends on the Pasquill stability class and it is shown in Tab. 1.

Table 1. Table summarizing the quantities used to set the ABL stability regimes identified with the six Pasquill stability classes (capital letters).

	A	B	C	D	E	F
$\bar{u}$ (ms^{-1})	1.5	2.5	4	4.5	3.5	2.5
u_* (ms^{-1})	0.1	0.17	0.25	0.26	0.16	0.09
L_{MO}^{-1} (m^{-1})	-0.14	-0.09	-0.03	0	0.03	0.14
n	0.07	0.07	0.1	0.015	0.35	0.55

RESULTS

In Fig. 1 are reported the vertical descriptors versus x/h, for a weakly convective ABL regime and for six emissions heights h_s/h. For each curve the long distance values approach to a common asymptotic value, regardless the source height. The *Sk* and the *Ku* plots show a nearly Gaussian symmetry already at short distances, this is particularly manifest in the high source emissions. On the other hand low source curves manifest a strong terrain influence.

It is possible to see the dependence of the maximum ground concentration on the source height in Fig. 2. The Gaussian approach shows a dependence of C_{max} from the square of source height. The two curves $(h_S/h)^{(-1)}$ and $(h_S/h)^{(-2)}$ allow a qualitative comparison of the GILTT results with the Brown et al. (1997) non Gaussian results where is shown that at the ground level $C_{max} \propto h_S^{-a}$ with $1 \le a \le 2$.

In Fig. 3 is shown a comparison between the GILTT standard deviations and the Briggs empirical curves (Arya, 1999). Four different source height h_s/h are considered. The plots show that at short horizontal distances from the source there is a reasonable agreement between curves, at large outsized discrepancies arise. An exception occur for the neutral case D, where the agreement reaches the best extent. It is known that empirical curves are drawn assuming Gaussian plume symmetry, furthermore these results depends on the choice of the ABL, which is set unlimited. This assumption is clearly unrealistic, nonetheless such empirical curves are extensively adopted when operative applications are concerned. A further remark regards the dependency on the source height h_s. Model results highlight their high susceptibility on s_s, feature not really clear when looking at the empirical curves.

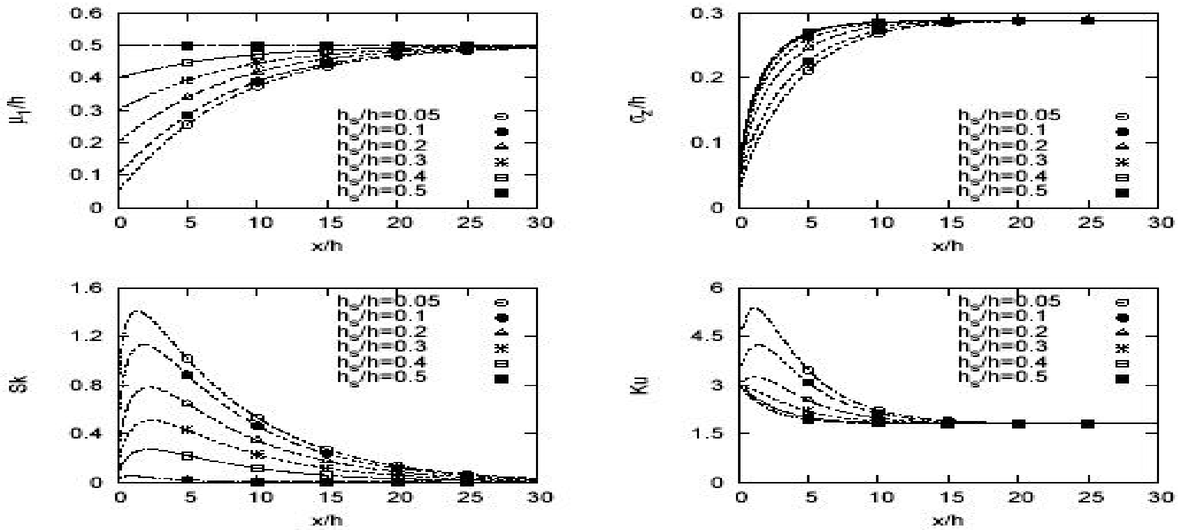

Fig. 1; Symmetries for the vertical distribution. Curves refer to six emission heights $h_S / h = 0.05, 0.1, 0.2, 0.3, 0.4, 0.5$*, ABL regime is C (see Tab. 1).*

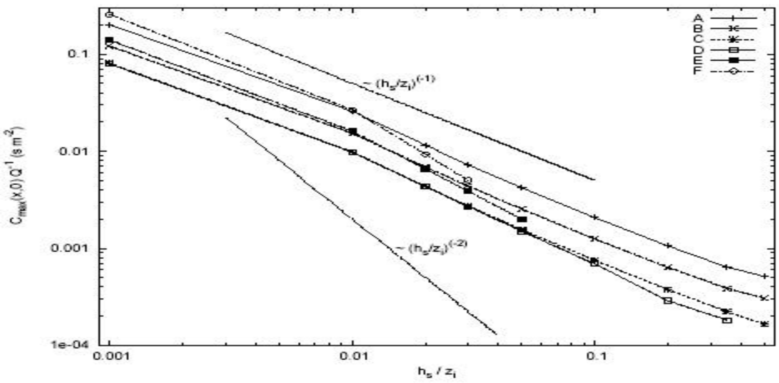

Fig. 2; Plots of $c_{\max}(x,0) \cdot Q^{-1}$ *versus the dimensionless distance* h_s/h*. The two curves* $\propto (h_s/h)^{(-2)}$ *and* $\propto (h_s/h)^{(-1)}$ *are also shown.*

CONCLUSIONS

Using an analytical solution of the two-dimensional steady ADE for a point source release, expressions for vertical plume symmetries have been derived. Moreover it was possible to easily evaluate the position and value of maximum ground concentration. Special emphasis has been devoted to $\overline{z}$ and s_z^2 (and s_s^2) because of their great operative concern.

The behaviour of the plume vertical standard deviation was outlined and compared with some very popular empirical ones, used in many operative air pollution models. It was outlined a general discrepancy occurs. It is evident that empirical formulae for s_s need to take into account the height of the source release h_s and the height of the ABL h. The formulae here presented can be useful for operative evaluation of atmospheric dispersion and a better understanding of advection-diffusion phenomena.

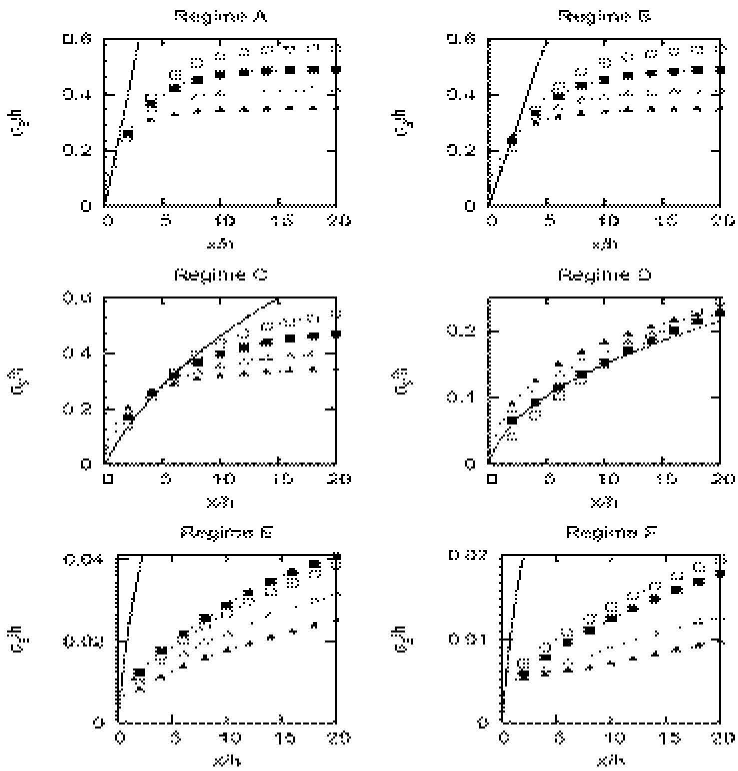

Fig. 3; Plots of s_s/h for the six stability classes of the ABL. For each class the s_s is evaluated for source height h_s/h = 0.01 (empty squares), 0.1 (black squares), 0.2 (empty triangles) and 0.3 (black triangles).

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