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A LAGRANGIAN DISPERSION MODEL WITH A STOCHASTIC EQUATION FOR THE TEMPERATURE FLUCTUATIONS.

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Abstract: The hybrid Lagrangian stochastic algorithm for buoyant plume rise from an isolated source described by Bisignano and Devenish (2015) is introduced into the Lagrangian dispersion model SPRAYWEB (Tinarelli et al, 1994, Alessandrini et al. 2013, Bisignano et al., 2017). In our approach each particle carries its own potential temperature, which evolves according to a stochastic differential equation as in Van Dop (1992). The buoyancy is calculated from the particle temperature and is directly included in the equation for the evolution of the velocity through a coupling term. We compare the concentration field simulated by the model with the results of a water tank experiment (Weil et al. 2002)

Key words: *Plume rise, Lagragian stochastic dispersion model, buoyancy, temperature fluctuations*

INTRODUCTION

Models of buoyant plumes originated with the work of Morton et al. (1956). These models describe the mean flow of the plume but do not explicitly take account of fluctuations in the velocity and buoyancy that occur within the plume (except through entrainment).

Several authors have attempted to model buoyant plume rise using a Lagrangian approach (e.g. Anfossi et al. 1993; Alessandrini et al. 2013). Here we consider a hybrid model introduced by Webster and Thomson (2002) in which the mean flow is calculated from a simple plume model and the fluctuations are calculated using an Lagrangian stochastic model (LSM). Webster and Thomson (2002) only considered fluctuations in the velocity and not the temperature; here we treat both fluctuations of the velocity and temperature. The governing equations for potential temperature and vertical velocity are derived from the Briggs (1984) plume equations. Then we separate the average and the turbulent fluctuating parts of the two variables through the application of the Reynolds decomposition. The final expressions of the stochastic differential equations (SDEs) for turbulent vertical velocity and potential temperature of the plume are obtained by adding terms of the form of Thomson (1987) to the turbulent fluctuating parts in order to satisfies the well-mixed condition. Accounting for fluctuations in temperature means that the Lagrangian particles carry its own potential temperature, which evolves according to a SDE as in Van Dop (1992). The effect of temperature fluctuations is directly included in the equation for the evolution of the velocity through a coupling term. To our knowledge, an expression for the temporal Lagrangian structure function for a passive scalar is not prescribed in literature. Hence, a completely satisfying approach for setting turbulence parameters for the temperature SDE based on Lagrangian description is not yet available. The constants required in the temperature SDE are set following the values commonly found in literature from both measurements and large eddy simulations (Devenish et al., 2010).

The above-described temperature SDE (Bisignano and Devenish, 2015) is introduced for the first time into the LSM SPRAYWEB (Tinarelli et al, 1994, Alessandrini et al. 2013, Bisignano et al., 2017) that, in its standard form, decribe the plume rise by use of the Anfossi et al. (1993) algorithm.

We validate the model against the water tank experiment of Weil et al. (2002).

THE PLUME RISE MODEL

The equations governing the rise of a buoyant plume in a uniform crossflow U are given by Briggs (1984):

$$\frac{d}{ds}(\pi v b^2) = E$$
$$(1+k_v)\frac{d}{ds}(\pi v w b^2) = \pi b^2 g' \qquad (1)$$
$$\frac{d}{ds}(\pi v g' b^2) = -N^2 \pi b^2 w$$

where $v=(U^2+w^2)^{1/2}$ is the velocity component along the plume axis, s is the distance along the plume axis, E is the entrainment rate (to be defined), w is the vertical velocity of the plume, b is the plume radius, N is the ambient buoyancy frequency and $g' = g(\theta(z)-\theta_a(z))/\theta_0$ is the reduced gravity in which $\theta(z)$ is the potential temperature of the plume at height z, $\theta_a(z)$ is the ambient potential temperature at height z and θ_0 is a reference temperature. The parameter k_v is the added-mass coefficient that accounts for the momentum of the ambient fluid displaced by the plume as the plume rises (here we considered a value of $k_v=1.3$ as suggested by Briggs, 1984). Equations (1) respectively describe the evolution of the volume flux V, momentum flux (per unit density) M and the buoyancy flux F. They are collectively known as the plume equations . First, we expand the left-hand side and express the plume equations in in term of w, b, and θ. Then we re-write the derivatives of w, b, and θ with respect to s as derivative with respect to $t=\int ds/v$. Making use of $N^2=g/\theta_0\, d\theta_a/dz = g/(\theta_0 w)\, d\theta_a/dt$ e get:

$$\frac{dw}{dt} = \frac{g(\theta-\theta_a)}{\theta_0} - E\frac{w}{b^2}$$
$$\frac{db}{dt} = \frac{E}{2\pi b} - \frac{bw}{2(1+k_v)v^2}\frac{g(\theta-\theta_a)}{\theta_0} + \frac{Ew^2}{2\pi b v^2} \qquad (2)$$
$$\frac{d\theta}{dt} = \frac{-E(\theta-\theta_a)}{b^2}$$

The equations (2) reduce to those of a vertically rising plume as $v \rightarrow w$ and to a bent-over plume as $w \rightarrow 0$. These equations are now used to calculate the mean velocity and temperature (which will be denoted by an overbar). The fluctuating velocity and temperature are denoted by a prime and will be calculated from SDEs. These are constructed from analogous equations to (2) and coupled with LSMs for w' and θ'. Let us show the application of a Reynolds decomposition to the equations for w and θ. Because equations (2) are linear in w and θ there are no second-order quantities and there is no feedback on the mean quantities by the fluctuating quantities. Let us assume, first, that $b'=0$ and hence that is evaluated in terms of the mean quantities alone and, second, that $E=E(w;v;U)$. Since we assume also that there are no fluctuations in θ_a , the Reynolds decompositions for w and θ are:

$$\frac{d}{dt}(w' + \overline{w}) = g\frac{(\overline{\theta}-\theta_a)}{\theta_0} + g\frac{\theta'}{\theta_0} - E\frac{\overline{w}}{b^2} - E\frac{w'}{b^2} \qquad (3)$$
$$\frac{d}{dt}(\overline{\theta}+\theta') = -E\frac{(\overline{\theta}-\theta_a)}{b^2} - E\frac{\theta'}{b^2}$$

Then the SDEs for w' and θ' are completed by adding respectively the LSM of Thomson (1987) for inhomogeneous turbulence and a LSM (in which for simplicity we assume that the temperature statistics are homogeneous) similar to that considered by van Dop (1992):

$$dw' = g\frac{\theta'}{\theta_0}dt - E\frac{w'}{b^2}dt - \frac{w'}{T_L}dt + \frac{1}{2}\left(\frac{1}{w} + \frac{w'}{\sigma_w^2}\right)d\,\sigma_w^2 + \sqrt{C_0\,\epsilon}\,dW$$
$$d\,\theta' = -E\frac{\theta'}{b^2}dt - \frac{\theta'}{T_\theta}dt - \frac{w'}{w}d\,\overline{\theta} + \sqrt{C_\theta\,\epsilon_\theta}\,dW_\theta \qquad (4)$$

where T_L and T_θ are the time scales on which w' and θ' decorrelates respectively, ε and ε_θ are the mean kinetic energy and scalar dissipation rates respectively, C_0 is the constant of proportionality in the second-order Lagrangian velocity structure function (we choose C_O=5), C_θ=1.6 is the Obukhov-Corrsin constant and σ_w is the vertical-velocity standard deviation. The third term of RHS of equation (5) for θ' arises from the time derivative of mean temperature that implicitly contains fluctuations of velocity. This term and the first term of RHS together ensure that θ is conserved following a particle, in both equations (4) the term involving E and the fluctuating quantities represent the effect of the entrainemnt on the turbulence, whereas the terms involving the time scales represent the internal turbulence of the plume. We neglect the covariance $\sigma_{w\theta}$ that may exist in reality.

The definition of entrainment, as in Bisignano and Devensih (2015), includes two additive entrainment mechanisms in a crosswind, one due to velocity differences parallel to the plume axis and the other due to velocity differences normal to the plume axis :

$$E = 2\pi b\left[\alpha\left(|w|\frac{|w|}{v}\right)m + \beta\left(|w|\frac{U}{v}\right)^m\right]^{1/m} \qquad (5)$$

We haveassumed that the difference between the horizontal component of the plume velocity and U is small relative to U. The constant coefficients α and β are associated with the two entrainment mechanisms: α with velocity differences parallel to the plume axis and β with velocity differences normal to the plume axis. We take $\alpha=0.1$ and $\beta = 0.5$ which are consistent with previous studies (Devenish et al., 2010). $m>1$ is a tunable parameter. Devenish et al. (2010) found that $m=3/2$ gave the best agreement with LES of buoyant plumes in a crosswind and field observations. We use this value throughout this study. The turbulence parameters σ_w, σ_θ, T_L and T_θ have been choosen to be related to the approriate mean quantities in the problem. Also it is necessary to limit the turbulence parameters in order to avoid numerical overflow in the oscillating region. We set:

$$\begin{aligned} \sigma_w &= \alpha\, max(|\overline{w}|, \overline{w}^*) \\ T_L &= \frac{b}{max(|\overline{w}|, \overline{w}^*)} \\ \sigma_\theta &= \gamma\, max(|\overline{\theta} - \theta_a|, \overline{\theta}^*) \\ T_\theta &= T_L \end{aligned} \quad \text{with} \quad \begin{aligned} \overline{w}^* &= 2^{-5/8}\pi^{-1/4}(6/5\,\alpha)^{-1}(9/10\,\alpha)^{1/2}F_0^{1/4}N^{1/4} \\ \overline{\theta}^* &= 2^{-5/4}\pi^{-1/4}(5/8\,\alpha)^{1/2}\theta_0 g^{-1}F_0^{1/4}N^{5/4} \\ b^* &= (2^{3/4} - 2^{3/8})\pi^{-1/4}(9/10\,\alpha)^{-1/2}F_0^{1/4}N^{-3/4} \end{aligned} \qquad (6)$$

where α and γ are tunable constants whose values are chosen equal to 0.1, and F_0 the initial buoyancy flux. The initialisation of *w, b* and *g′* (denoted with a subscript 0) for a pure plume whose initial buoyancy flux is known is not straightforward. We estimate w_0 by equating the initial radius $b_0 = 2z$ so that $w_0 = (b_0\, g'_0)^{1/2}$. Since the initial buoyancy flux $F_0 = \pi b_0^2\, g'_0\, v_0$ we obtain a cubic polynomial for either w_0 or g'_0 for given b_0, whose roots can be inferred by analysing the discriminant Δ. Any of the three cases ($\Delta>0$, $\Delta=0$, $\Delta<0$) will produce a physical solution. In the case that there are three real roots ($\Delta > 0$), shows that two of these roots will be negative, and can thus be discarded (Bisignano and Devenish (2015) for further details).

THE CASE STUDY.

We considered the Weil et al. (2002) experiment, which investigate the plume dispersion in the convective boundary layer (CBL) using a convection tank. The focus is on highly-buoyant plumes that become trapped in the CBL capping inversion and resist downward mixing. Such plumes are defined by a dimensionless buoyancy flux $F_*>0.1$. F_* is defined as:

$$F_* = \frac{F}{U\,w_*^2 z_i} \qquad (9)$$

where F_b is the stack buoyancy flux, U is the mean wind speed, w_* is the convective velocity scale, and z_i is the CBL depth. In partuclar the experiment is characterized by different values of the normalized stack buoyancy flux and it reproduces all components of the lateral and vertical dispersion parameters (rms meander, relative dispersion, total dispersion), mean and root-mean-square concentration fields as a function of F_* for continuous buoyant releases.

RESULTS

We focus on mean height, horizontal and vertical plume standard deviations, for the cases $F_*=0.1$, $F_*=0.2$, and $F_*=0.4$. We compare the measured and the simulated results, both with Anfossi et al. (1993) plume rise and Bisignano and Devenish (2015) plume rise. The Figures 1–3 (that refers to the cases $F_*=0.1$, $F_*=0.2$, and $F_*=0.4$ respectively) show that the model plume characteristics agree very well with the data plume characteristics for a wide range of normalized buoyancy flux values. This preliminary results show that the model is able to correctly reproduce the basic behaviour of the plume rise phenomenon in convective conditions, though with a little overestimation for the vertical standard deviation and a little underestimation in the mean height. We found that the vertical spread, evaluated with the above-described plume rise, matches the data better than that evaluated with the Anfossi et al. (1993) plume rise, characterised by the absence of temperature fluctuations.

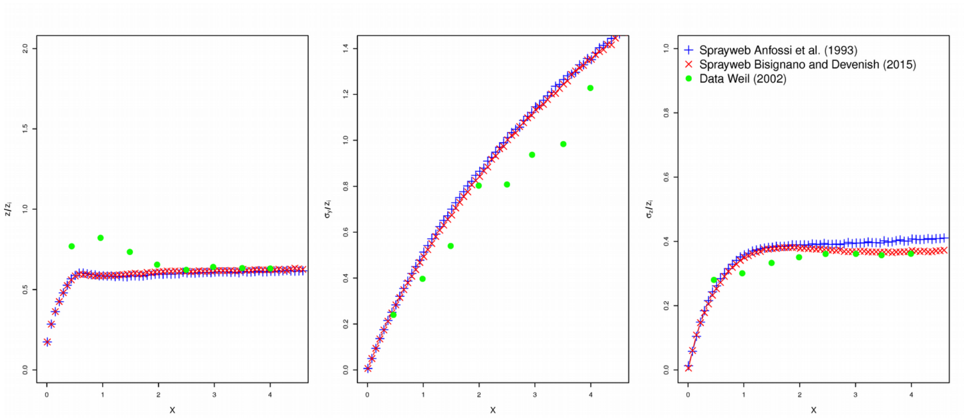

Figure 1. Comparison of the measured (Weil et al. 2002) and simulated (both with Anfossi et al. (1993) and Bisignano and Devenish (2015) plume rises) mean height, horizontal and vertical standard deviations for $F_*=0.1$.

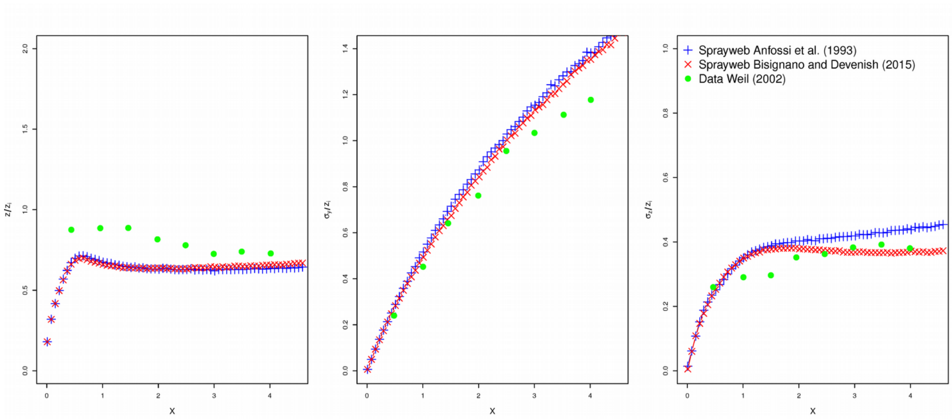

Figure 2. As in Figure 1 but for $F_*=0.2$

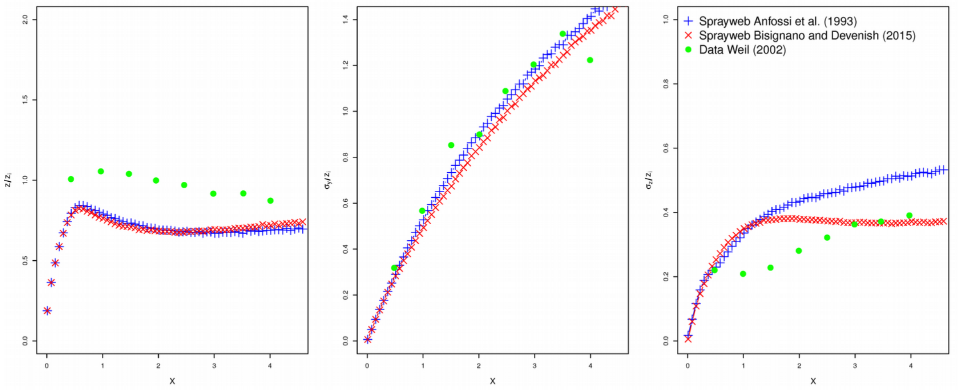

Figure 3. As in Figure 1 but for F_{*}=*0.4*

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