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Bayesian transdimensional inverse reconstruction of the ^{137}Cs Fukushima-Daiichi release

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Abstract:
The accident at the Fukushima-Daiichi nuclear power plant resulted in massive and rapidly changing releases of radionuclides into the atmosphere. Using a relevant set of measurement data, the dynamics of the releases can be assessed using advanced inverse modelling techniques. A Bayesian inversion is particularly suitable as it allows a rigorous modelling of statistical errors and to easily incorporate information of different nature in the reconstruction of the source and the associated uncertainties. We implement the Reversible-Jump MCMC algorithm, a sampling technique capable of reconstructing the magnitude distribution of the Fukushima-Daiichi ^{137}Cs source as well as its temporal discretisation.

We obtain a posteriori distributions assessing the magnitude and temporal evolution of the ^{137}Cs release between 11 and 24 March, and a quantification of the uncertainties associated with the observations and the model. The total reconstructed activity released is estimated to be between 10 and 20 PBq. Finally, the adaptive discretisation of the source term gives an almost hourly profile over some intervals of high temporal variability.

Key words: *Inverse modelling, Bayesian inference, MCMC methods, Release assessment, Source term, Fukushima-Daiichi, Cesium 137*

Introduction

The accident at the Fukushima-Daiichi power plant, caused by a tsunami off the coast of Japan on 11 March 2011, resulted in a large atmospheric release of radionuclides for several weeks. The assessment of such a release is difficult and involves considerable uncertainties. Reconstructing the temporal evolution of the ^{137}Cs source means determining a highly variable release rate over several hundred hours.

Bayesian inverse methods have proven to be effective in estimating radionuclide sources and associated uncertainties (Dumont Le Brazidec et al., 2021). In the Bayesian framework, the crucial elements defining the source $\mathbf{x}$ are the vector of the release rates q_i (each q_i is a constant release rate on a time window and the release function is represented by this discretised release), and hyperparameters such as those describing the scale matrix of observation errors $\mathbf{R}$ or the time windows of the release rates.

Using the Bayes' rule, the probability density function (pdf) of this source $\mathbf{x}$ given a set of observations $\mathbf{y}$ is written as:

$$p(\mathbf{x}|\mathbf{y}) = \frac{p(\mathbf{y}|\mathbf{x})p(\mathbf{x})}{p(\mathbf{y})} \propto p(\mathbf{y}|\mathbf{x})p(\mathbf{x}) \tag{1}$$

where the first term $p(\mathbf{y}|\mathbf{x})$ quantifies the fit of $\mathbf{x}$ to the data and is called the likelihood. To quantify this fit, a set of modelled concentrations corresponding to the observed concentrations is built from the source $\mathbf{x}$ with the help of an observation operator $\mathbf{H}$. This observation operator represents the transport of the radionuclides and is assumed linear with respect to the release rates $\mathbf{q}$. The likelihood used in this study is a log-Cauchy distribution. The second term, $p(\mathbf{x})$ is the pdf of the source before access to observations and is called the prior. The pdf $p(\mathbf{x}|\mathbf{y})$, once defined, can be reconstructed with sampling techniques such as Markov chain Monte Carlo (MCMC) methods (Dumont Le Brazidec et al., 2020).

In the case of ^{137}Cs releases from Fukushima-Daiichi, the definition of the appropriate time windows defining the discretisation of the release function is not obvious. This representation depends on the case studied: given the set of observations, there is more or less information available to define a more or less time resolved source term. More precisely, a large amount of informative data over a time interval should yield small time windows describing **q** over this time interval, while little information should lead to large time windows. This results in an irregular time discretisation of the source term. The releases of ^{137}Cs from the Fukushima-Daiichi nuclear power plant are very large over several weeks, and with an important temporal variability. The choice of discretisation is a crucial and difficult task.

Fukushima-Daiichi ^{137}Cs release modelling

The dataset used consists of $14,248$ air concentration measurements from 105 stations taken over the Japanese territory. The locations of the stations are shown in Figure 1. The atmospheric dispersion of the

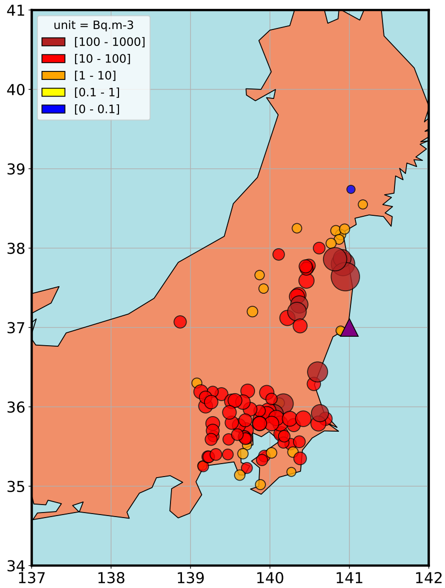

Figure 1: Maximum ^{137}Cs air concentrations for each station in Bq.m^{-3}. The Fukushima-Daiichi NPP is represented by the purple triangle.

^{137}Cs plume is simulated using the three-hourly OPER OD ECMWF fields and the ldX Eulerian model, a model of the IRSN C3X operational platform, validated in particular on the ^{106}Ru releases in 2017 (Saunier et al., 2019).

Reversible-Jump MCMC

To retrieve the irregular time discretisation of the source term, we propose the use of the Bayesian Reversible-Jump MCMC algorithm Bodin and Sambridge (2009); Liu et al. (2017). The RJ-MCMC algorithm acts as a natural extension of the very popular Metropolis-Hastings (MH) MCMC algorithm to transdimensional discretisation grids. The RJ-MCMC allows to sample a source with no assumption (other than a prior probability) on the time windows defining the constant release rates. These time intervals are not fixed in advance and are sampled at the same time as the other variables. This makes it possible to find the distribution of these time intervals, and thus to calculate the best definition of the temporal evolution of the source and the associated uncertainties. Technically, the RJ-MCMC is a MH algorithm with transdimensional random walks such as a process of birth of a new time window (which increases the number of constant release rates, thus variables, thus the dimension of the source **x**).

Application to the Fukushima-Daiichi ^{137}Cs release

The temporal evolution of the median release rate with its associated standard deviation, reconstructed with the help of the RJ-MCMC, is presented on Figure 2. This time evolution is compared to the source term of Saunier et al. (2013) which was estimated using gamma dose rate measurements, but based on the same meteorological data and transport model, as well as to the more recent source term of Terada et al. (2020). The source term reconstructed shows a convincing fit to the Saunier et al. (2013) and Terada et al.

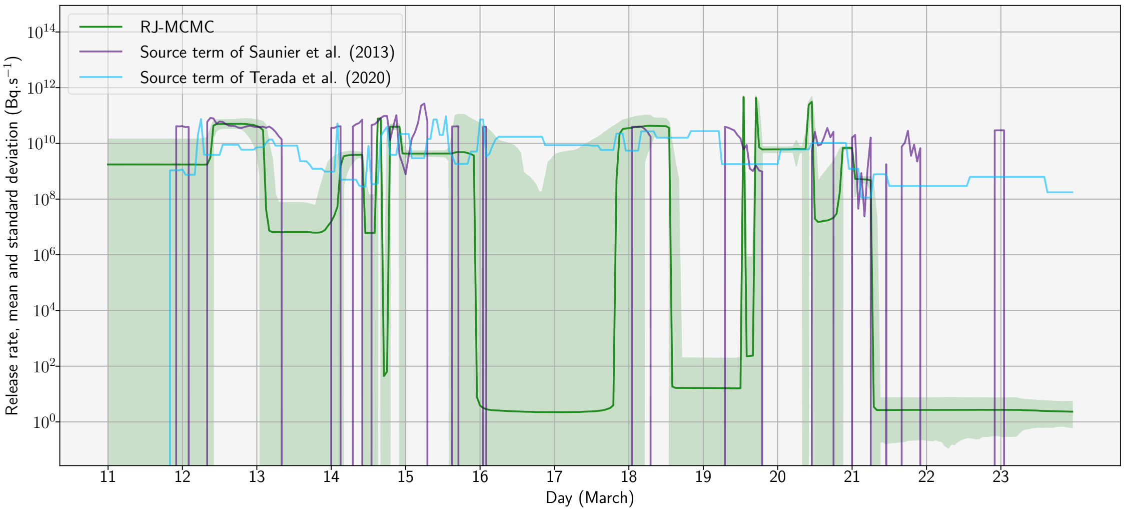

Figure 2: Evolution of the ^{137}Cs source release rate in Bq.s^{-1} reconstructed using the RJ-MCMC algorithm. The green line corresponds to the sampled median release rate while the light green area corresponds to the area between $\mu_{\ln q}$ the median and $\sigma_{\ln q}$ the standard deviation of the hourly release rates. A comparison to the Saunier et al. (2013) and Terada et al. (2020) source terms is provided.

(2020) source terms, since differences below 10^9 Bq.s^{-1} can be neglected. Various important release peaks can be observed with a magnitude approaching or exceeding 10^{11} Bq.s^{-1}. The MCMC samples periods of high temporal variability in the release rate (such as between 19 and 21 March), and low temporal variability (such as between 11 and 14 March). The reconstruction of this high variability of the release rate variability is made possible by the RJ-MCMC transdimensional algorithm: some time windows are accurately evaluated while others are coarsely reconstructed.

Figure 3 describes several quantities related to the same reconstruction provided by the RJ-MCMC. The histogram of the relative differences in $\log_{10}$ between the median predictions and the observations is provided in panel a. A good match between observations and predictions is observed, although the observations are globally underestimated by the predictions. The FAC2 and FAC5 scores reach 0.321 and 0.68, respectively.

The total release pdf is given in panel b. Most of the total release distribution mass is confined between 10 and 20 PBq, peaking at 14 PBq. This range is consistent with the literature, which estimates the ^{137}Cs total release to be between 5 and 30 PBq.

Histogram c and d present measures of the complexity provided by the RJ-MCMC algorithm. A boundary is a separation time step between two time windows where the release rate is modelled as constant. The histogram of the number of boundaries (which relates to the general complexity of the modelling) is provided in panel c: 45 boundaries are sampled in average for a maximum number of 313. On panel d is shown the average number of sampled boundaries at each time with the associated standard deviation. The y-axis corresponds to the number of boundaries counted around a given hour t. It is:

$$\mathrm{E}[\mathrm{Card}\{\lambda_i \in [t-5\ldots t+5] | \lambda_i \in (\lambda_0, \ldots, \lambda_{\mathrm{N_{imp}}})\}] \tag{2}$$

the average over all samples of the number of boundaries counted around t and represents the number of boundaries assessed as necessary by the RJ-MCMC algorithm to model the release over a given time interval (around t). It can be seen as a measure of the variability of the release. A large variability can be observed between 14 and 15 March, and between 19 and 21 March, correlating very well with results of Figure 2.

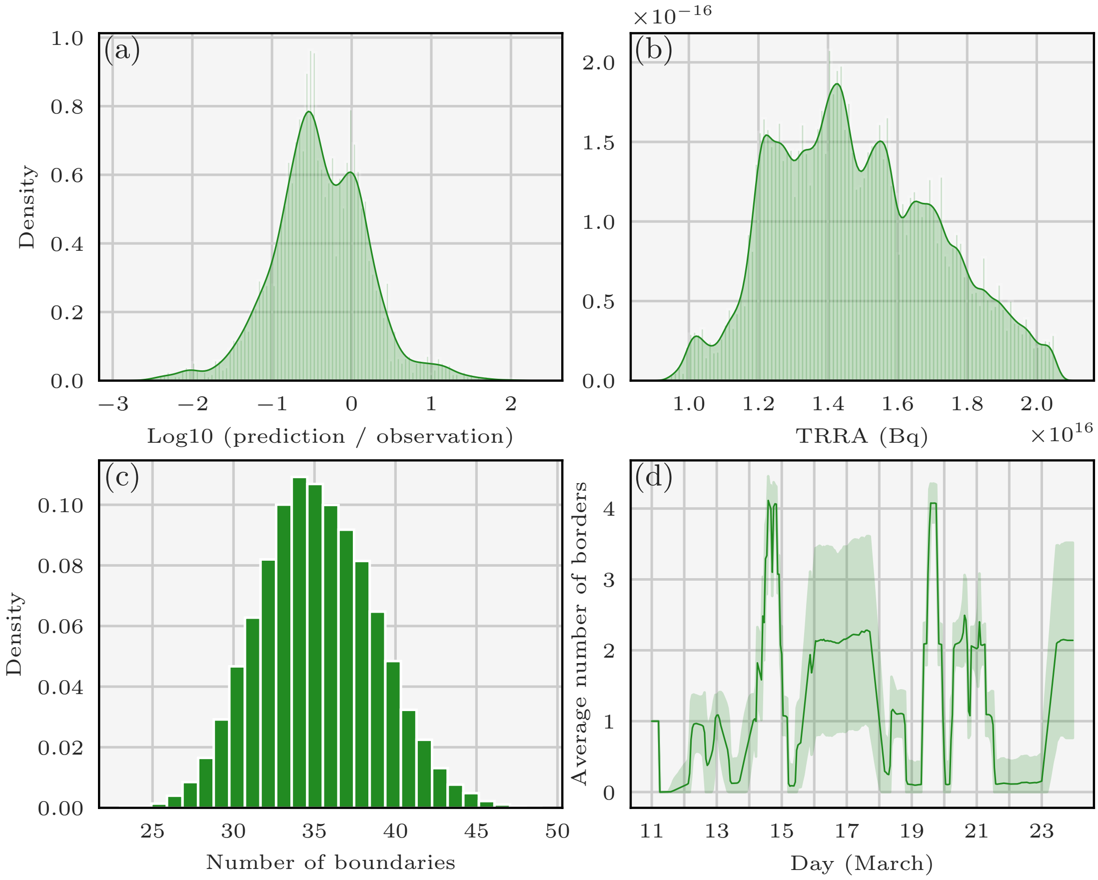

Figure 3: Densities or averages of ^{137}Cs source variables sampled using the RJ-MCMC algorithm.
(a) Relative differences in $\log_{10}$ between observations and predictions histogram. (b) Histogram of the ^{137}Cs Total Retrieved Released Activity (TRRA) in Bq. (c) Density of the number of boundaries. (d) Mean ($\pm$ standard deviation) of the number of boundaries around hours.

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